Reflection Method

- ***Seismic Reflection Travel-Times*** have eqns of a hyperbola. For single layer over halfspace,

$$t^2 = \frac{x^2}{V_1^2} + \frac{4h_1^2}{V_1^2}$$

has ***intercept*** $2h_1/V_1$ and ***slope of the asymptote*** is $1/V_1$!

- For two layers:

$$\frac{(t-t_0)^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow a = 2\left(\frac{h_1}{V_1} + \frac{h_2}{V_2}\right); \frac{b}{a} = \frac{1}{V_2}; t_0 = \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

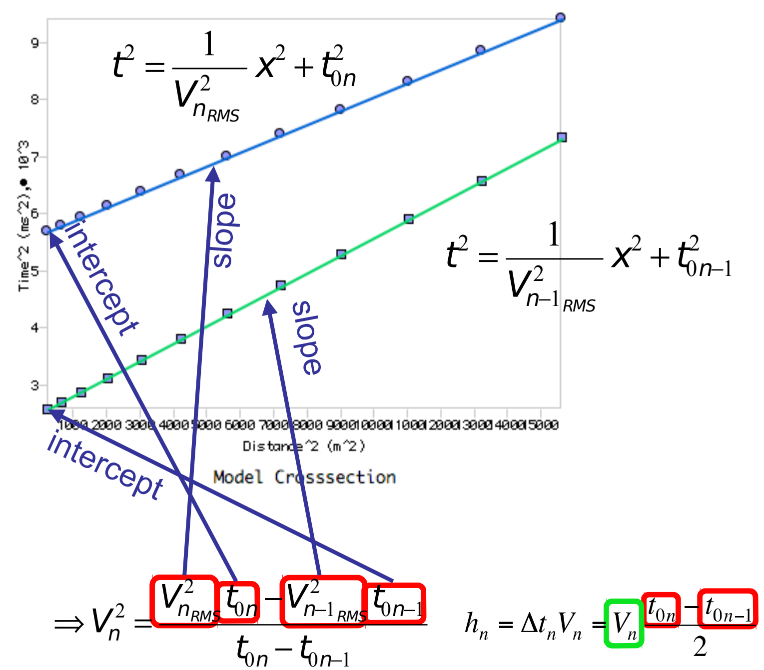
- For a dipping layer,

$$t^2 = \frac{x^2}{V_1^2} - \frac{4h_1\sin(\alpha)}{V_1^2}x + \frac{4h_1^2}{V_1^2}$$

- ***Dix Equations*** multiple layers:

$$V_{n_{RMS}}^2 \cong \frac{\sum_{i=1}^{n} V_i^2 \Delta t_i}{\sum_{i=1}^{n} \Delta t_i}$$

$$V_n^2 = \frac{V_{n_{RMS}}^2 t_{0n} - V_{n-1_{RMS}}^2 t_{0n-1}}{t_{0n} - t_{0n-1}}$$



- ***Normal Move-Out (NMO)***: reflection travel-time at distance x minus t at $x = 0$:

  * For single layer case:

$$T_{NMO} = t - t_0 = \frac{\sqrt{x^2 + 4h_1^2}}{V_1} - \frac{2h_1}{V_1} \qquad t = \frac{\sqrt{x^2 + 4h_1^2}}{V_1} = \sqrt{\frac{x^2}{V_1^2} + \frac{4h_1^2}{V_1^2}} = t_0\sqrt{1 + \frac{x^2}{V_1^2 t_0^2}}$$

  * First-order binomial series approximation:

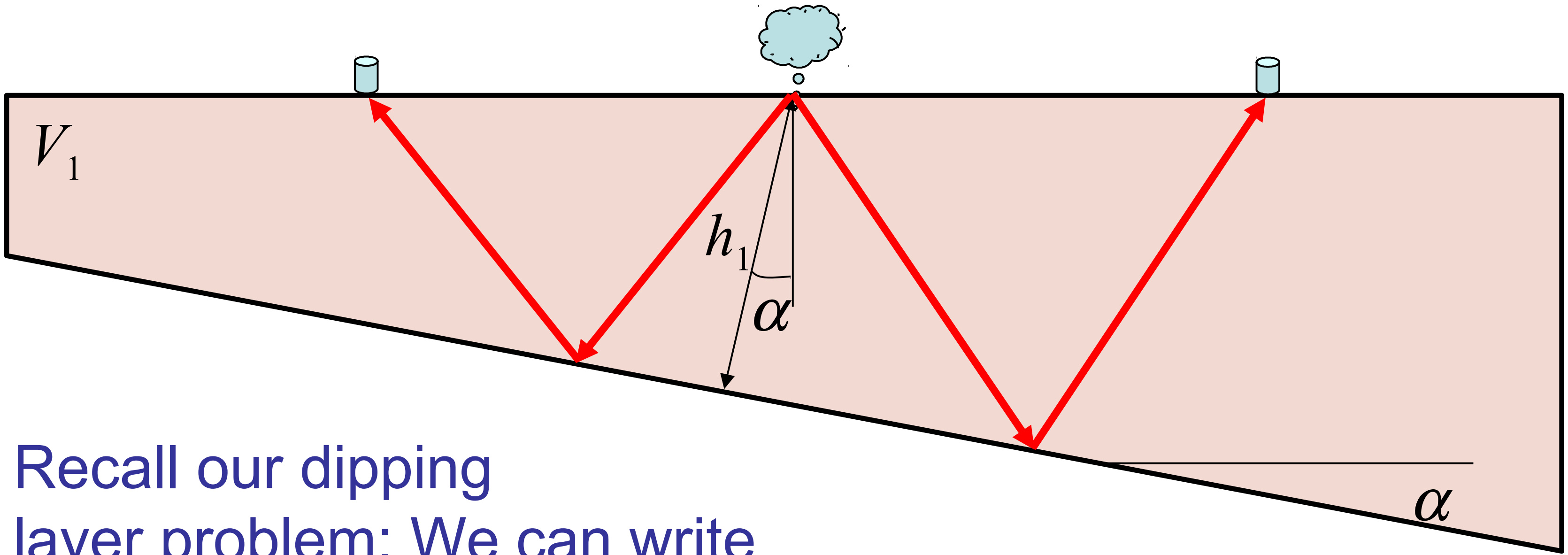
$$T_{NMO} \doteq \frac{x^2}{2t_0V_1^2}$$

(& second-order): $T_{NMO} \doteq \frac{x^2}{2t_0V_1^2} - \frac{x^4}{8t_0^3V_1^4}$

(Useful to write travel-time in terms of only V & observed t_0)

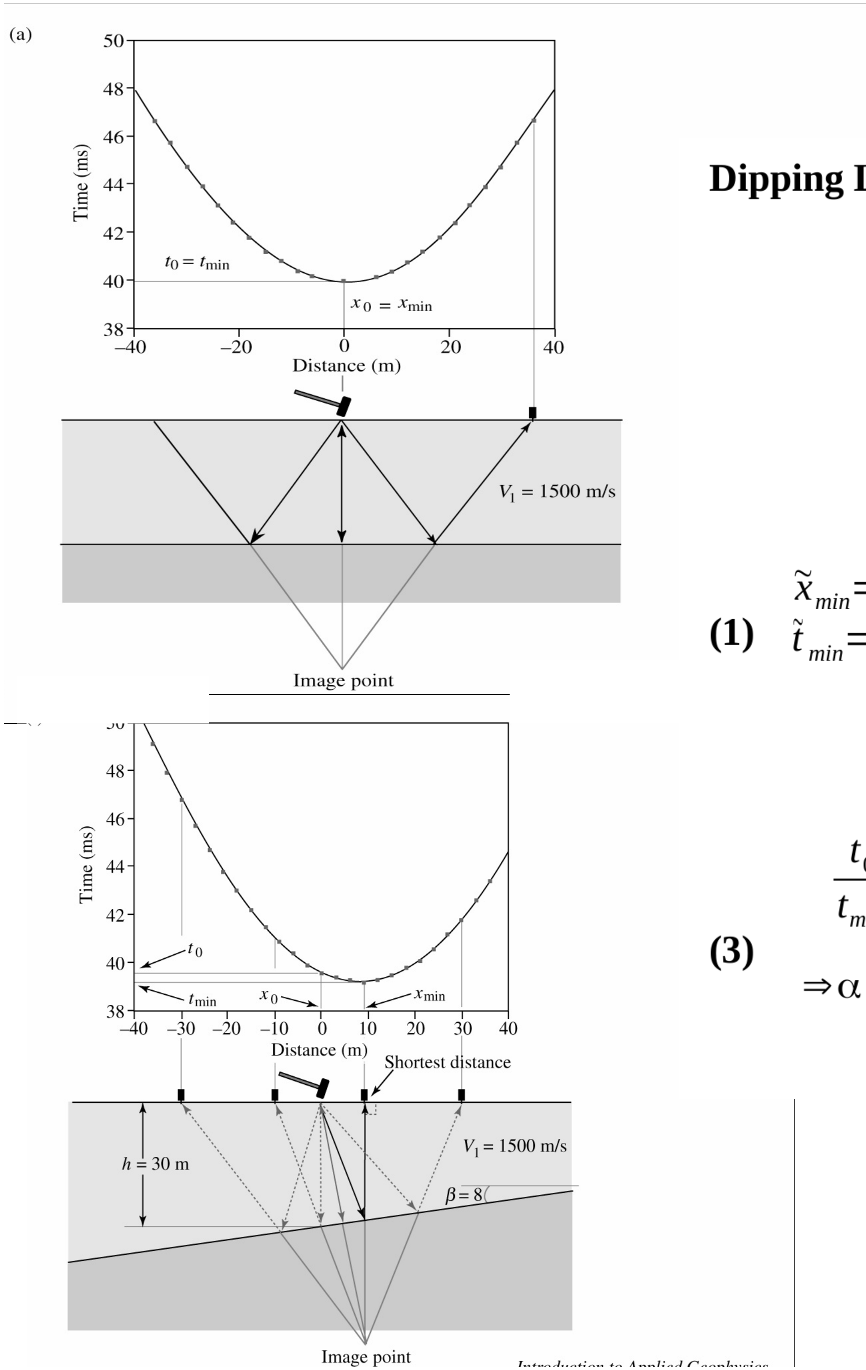
  * Can also be applied to multiple layers using the ***Dix equation*** approximation for V_{rms}:

$$T_{NMO} = \frac{x^2}{2t_0V_{rms}^2}$$



Recall our dipping layer problem: We can write

$$t = \frac{\sqrt{x^2 + 4h_1^2 - 4h_1 x \cos\left(\alpha + \frac{\pi}{2}\right)}}{V_1} = \frac{\sqrt{4h_1^2 + x^2 - 4h_1 x \sin\alpha}}{V_1}$$



Dipping Layer t-x equation: Minimum x, t

$$\tilde{t}^2 = \tilde{x}^2 - 2\tilde{x}\sin(\alpha) + 1$$

$$\tilde{t} = \frac{t}{t_0} \quad \Leftarrow \quad t_0 = \frac{2h'}{V} \qquad \tilde{x} = \frac{x}{2h'}$$

(1) $$\tilde{x}_{min} = \sin(\alpha) \qquad \tilde{t}_{min} = \cos(\alpha)$$

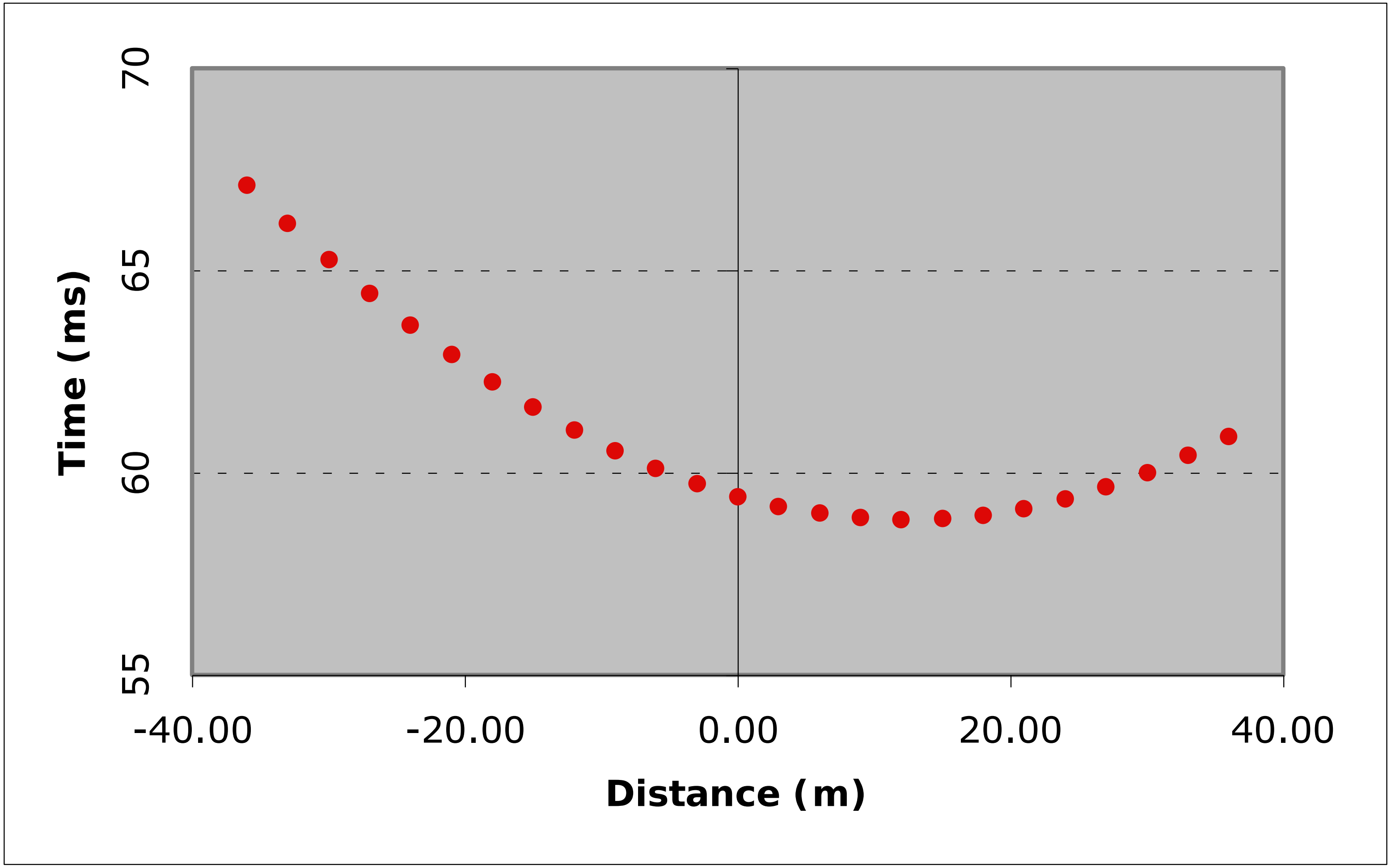
(2) $$x_{min} = 2h'\sin(\alpha) \qquad t_{min} = \frac{2h'\cos(\alpha)}{V} \qquad t_0 = \frac{2h'}{V}$$ → **from t-x plot**

(3) $$\frac{t_0}{t_{min}} = \cos(\alpha) \quad \Rightarrow \alpha = \cos^{-1}\left(\frac{t_0}{t_{min}}\right)$$

and

(4) $$h' = \frac{x_{min}}{2\sin(\alpha)} \quad (a) \qquad h = \frac{h'}{\cos(\alpha)} \quad (b)$$

Example: $V_1 = 1500$, $h = 45$ m, $\alpha = 8°$



Helpful Hint: *Reflect* does not model dipping layers, but the Table 4-6 Excel spreadsheet does!

Using the same approach used earlier for a horizontal layer,

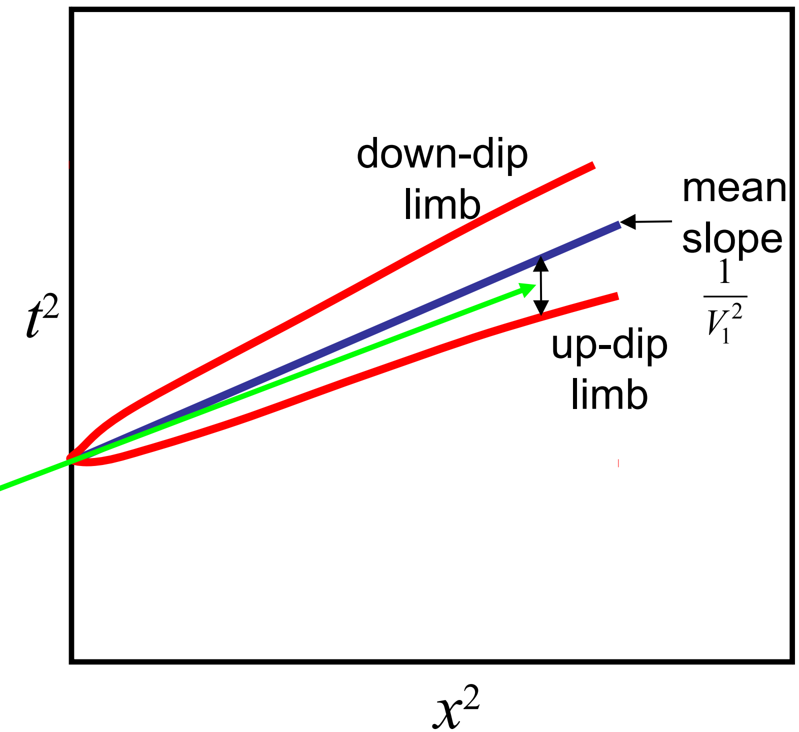
$$t = \frac{\sqrt{4h_1^2 + x^2 - 4h_1 x \sin\alpha}}{V_1} = \sqrt{\frac{4h_1^2}{V_1^2}\left(1 + \frac{x^2 - 4h_1 x \sin\alpha}{4h_1^2}\right)} = t_0\sqrt{1 + \frac{x^2 - 4h_1 x \sin\alpha}{4h_1^2}}$$

can be expanded and truncated as

$$t \doteq t_0\left(1 + \frac{x^2 - 4h_1 x \sin\alpha}{8h_1^2}\right) = t_0 + \frac{x^2 - 4h_1 x \sin\alpha}{h_1 V_1}$$

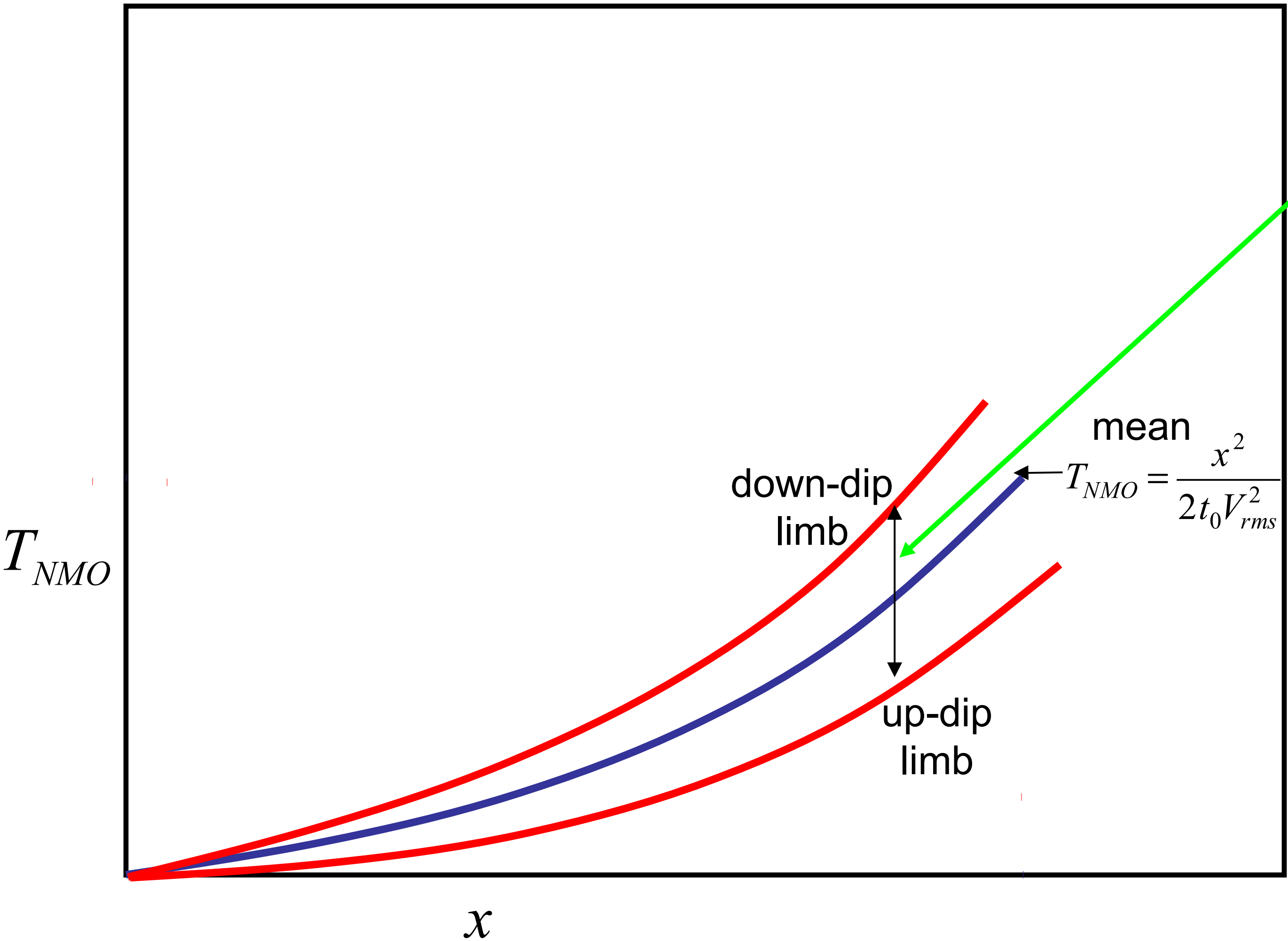
Recall the $x^2 - t^2$ plot for a dipping layer: Could average $\left(t_{+x}^2 + t_{-x}^2\right)/2$ to get a line with slope $1/V_1^2$, and the difference between the line and the limbs was related to the dip angle:

$$t^2 = \frac{x^2}{V_1^2} - \frac{4h_1 \sin\alpha}{V_1^2}x + \frac{4h_1^2}{V_1^2}$$



Binomial series approximation:

$$t \doteq t_0 + \frac{x^2 - 4h_1 x \sin\alpha}{4h_1 V_1} \qquad t_{+x} - t_{-x} \doteq \frac{-4h_1 x \sin\alpha}{4h_1 V_1} - \frac{-4h_1(-x)\sin\alpha}{4h_1 V_1} = -\frac{2x\sin\alpha}{V_1} \equiv T_{DMO}$$



We can similarly use the difference between up-dip & down-dip NMO to estimate dip via:

$$\alpha = \sin^{-1}\left(-\frac{V_1 T_{DMO}}{2x}\right)$$