Reflection Method

- ***Seismic Reflection Travel-Times*** have eqns of a hyperbola. For single layer over halfspace,

$$t^2 = \frac{x^2}{V_1^2} + \frac{4h_1^2}{V_1^2}$$

has ***intercept*** $2h_1/V_1$ and ***slope of the asymptote*** is $1/V_1$!

- ***Normal Move-Out (NMO)***: reflection travel-time at distance x minus t at $x = 0$:

$$t_{NMO} = \frac{\sqrt{x^2 + 4h_1^2}}{V_1} - \frac{2h_1}{V_1}$$

- For two layers, algebra starts to get complicated:

$$\frac{(t - t_0)^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow a = 2\left(\frac{h_1}{V_1} + \frac{h_2}{V_2}\right); \frac{b}{a} = \frac{1}{V_2}; t_0 = \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

- For a dipping layer,

$$t^2 = \frac{x^2}{V_1^2} - \frac{4h_1 \sin(\alpha)}{V_1^2} x + \frac{4h_1^2}{V_1^2}$$

Getting velocity structure: The $x^2 - t^2$ method

If we have travel-times from a reflection, can plot t^2 vs x^2 to get parameters of thickness & velocity from slope and intercept of the resulting line fit!

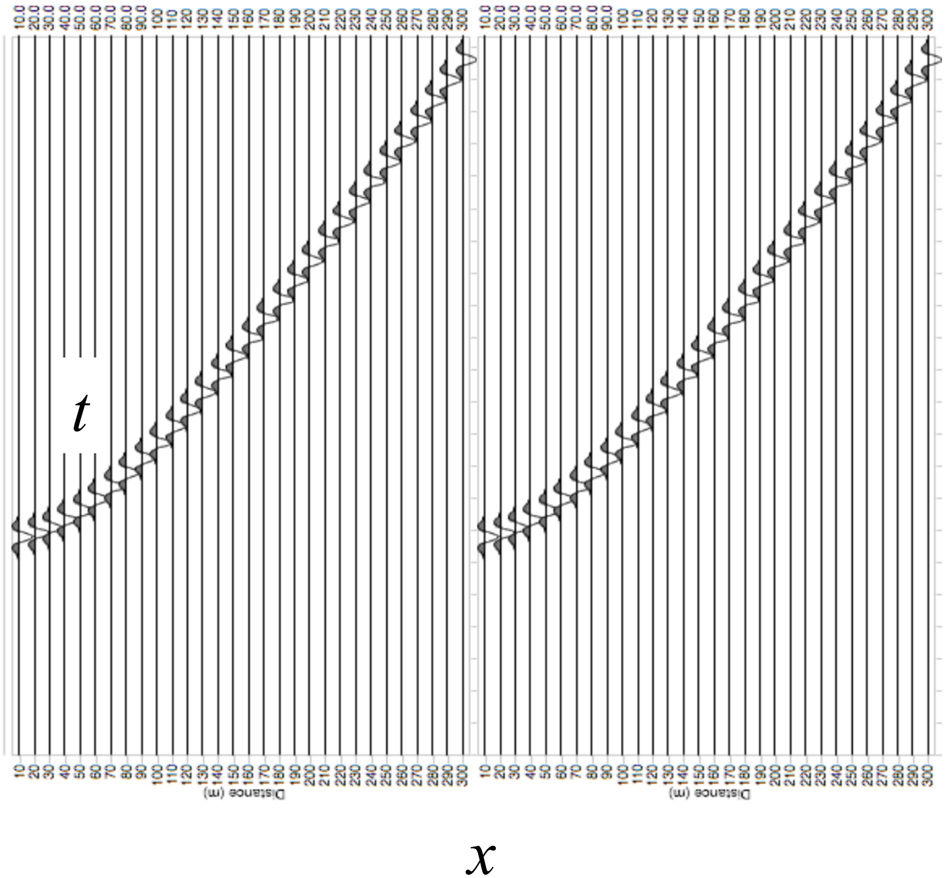
Recall that the goal of geophysics is to ***invert for parameters*** (in this case, velocity and thickness)

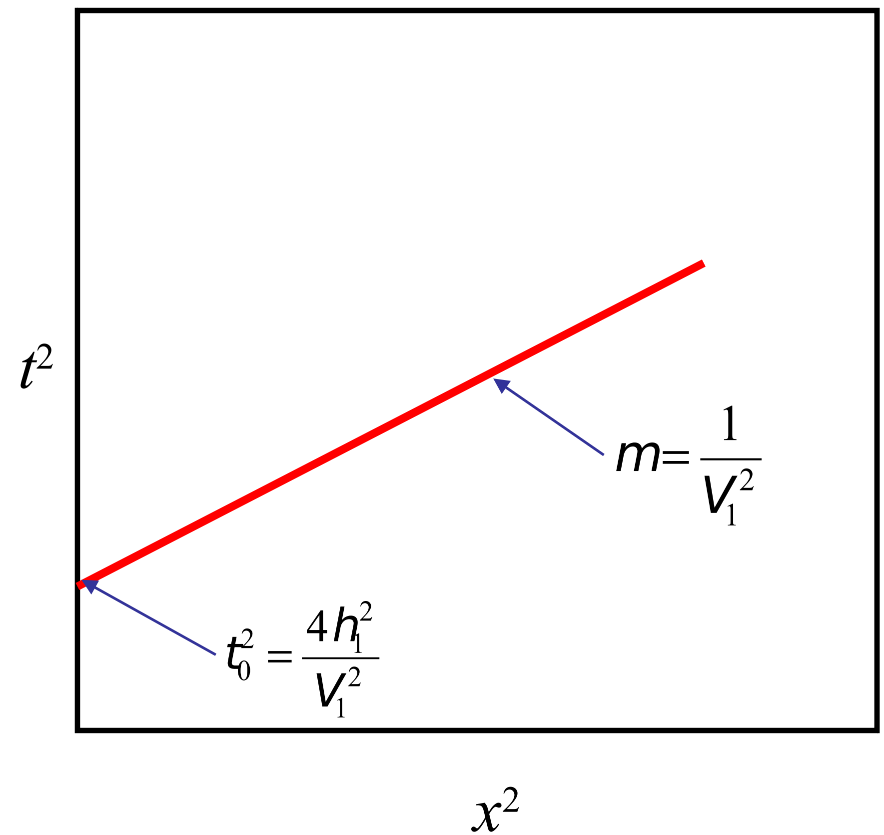
Given travel-times from a reflection, can plot t^2 vs x^2 to get parameters of thickness & velocity from slope and intercept of the resulting line fit.

For the single layer case:

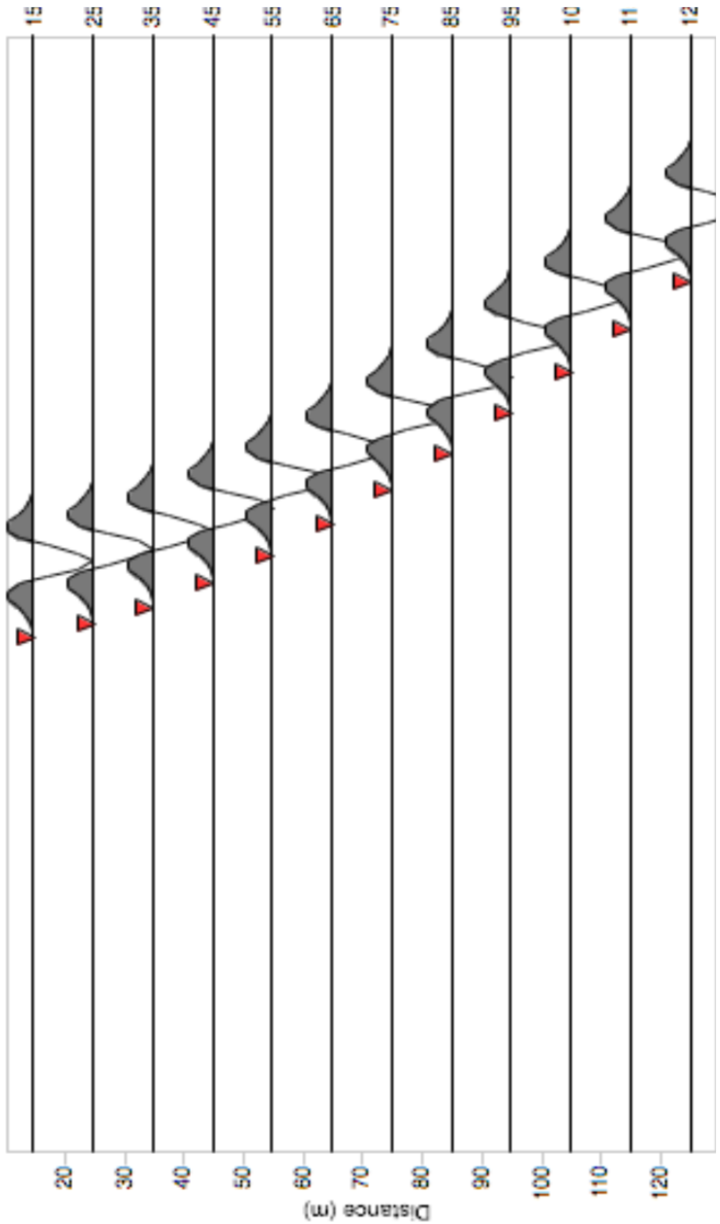
$$t = \frac{\sqrt{x^2 + 4h_1^2}}{V_1} \quad \Rightarrow \quad t^2 = \frac{1}{V_1^2}x^2 + 4\frac{h_1^2}{V_1^2}$$

So for slope m, intercept t_0^2 of a line fit:

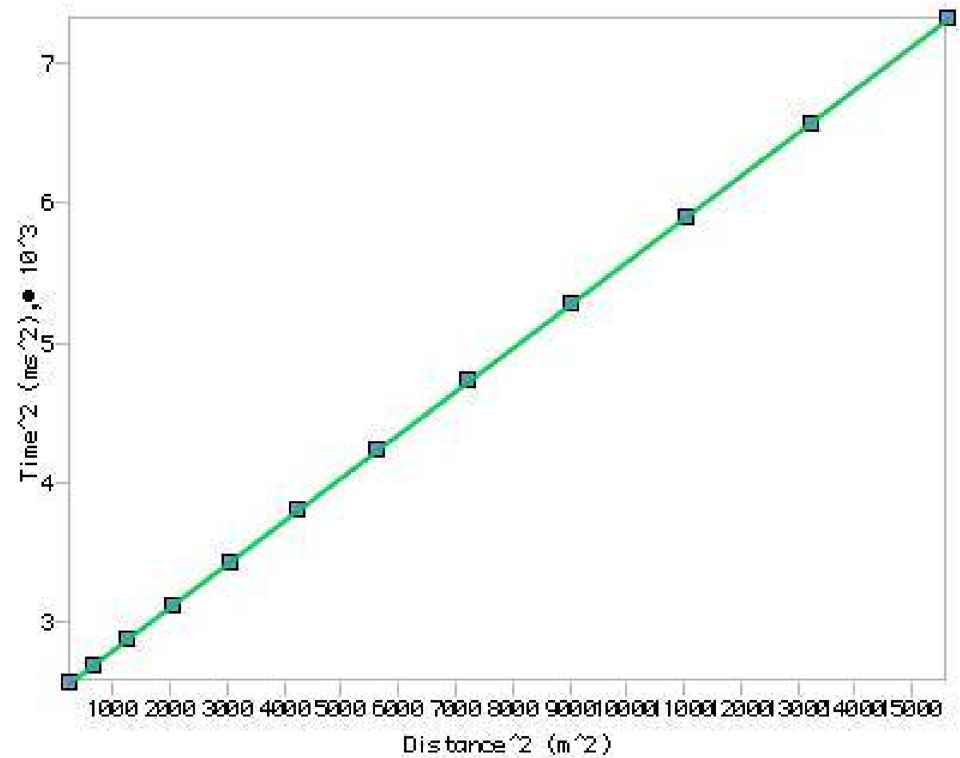




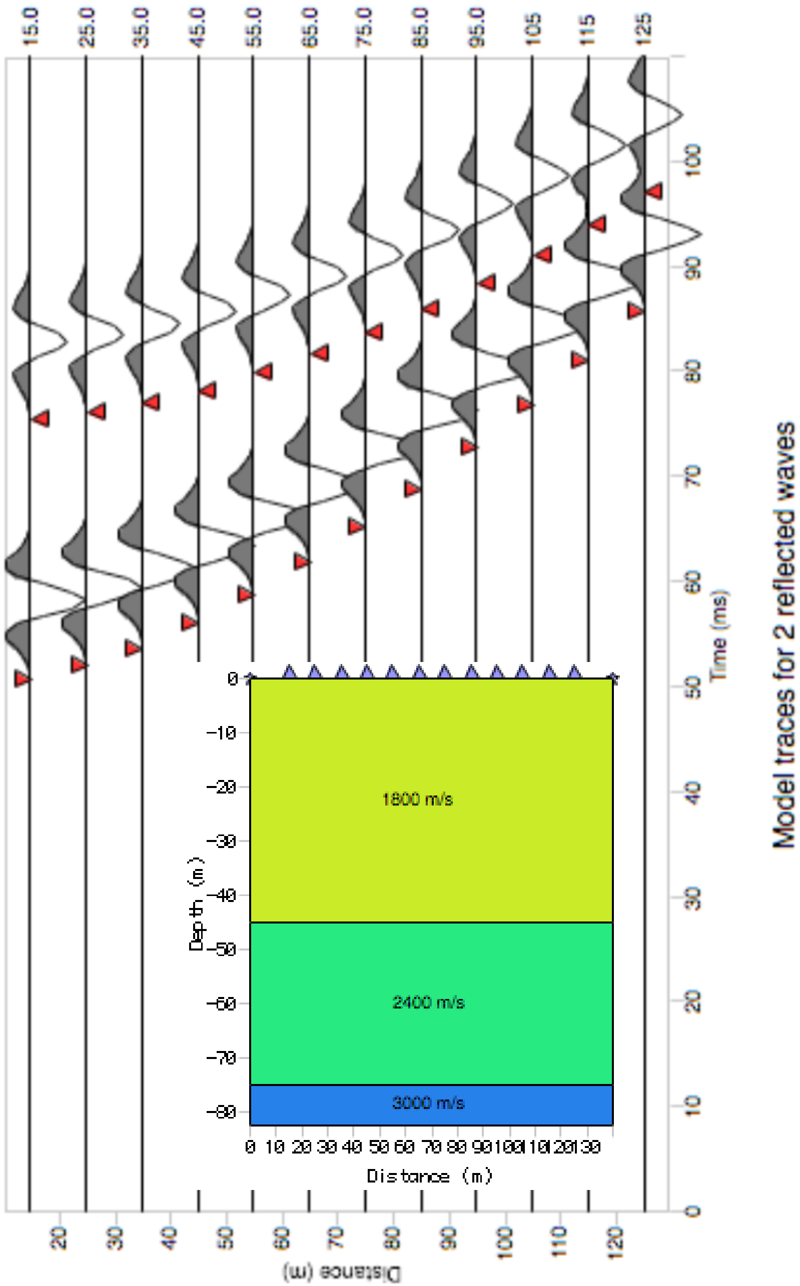
$$V_1 = \frac{1}{\sqrt{m}} \qquad h_1 = \frac{t_0 V_1}{2}$$



Note the single-layer approximation is the approach taken to inverting data in *Reflect*!

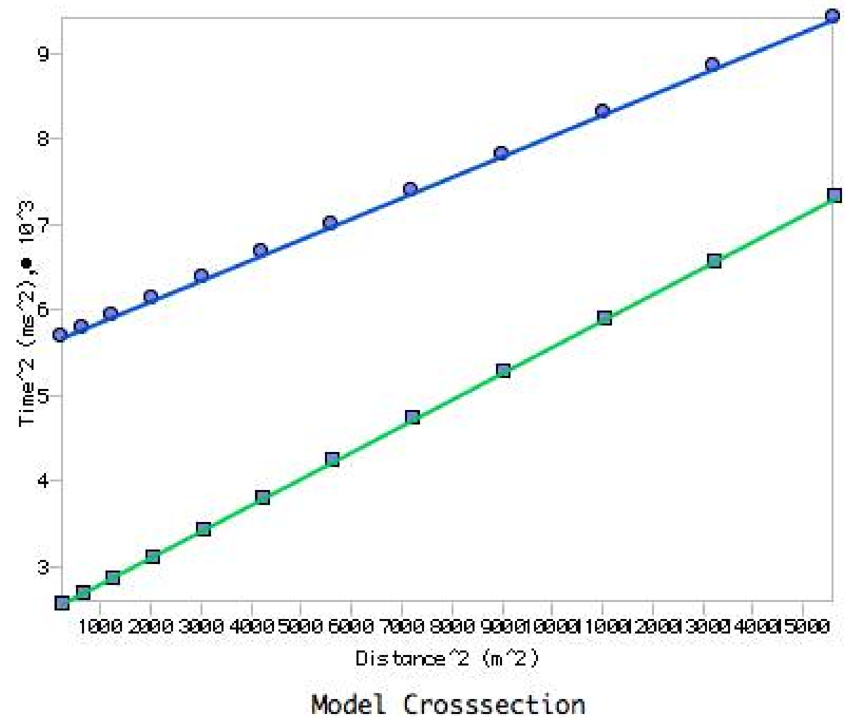


Model Crosssection



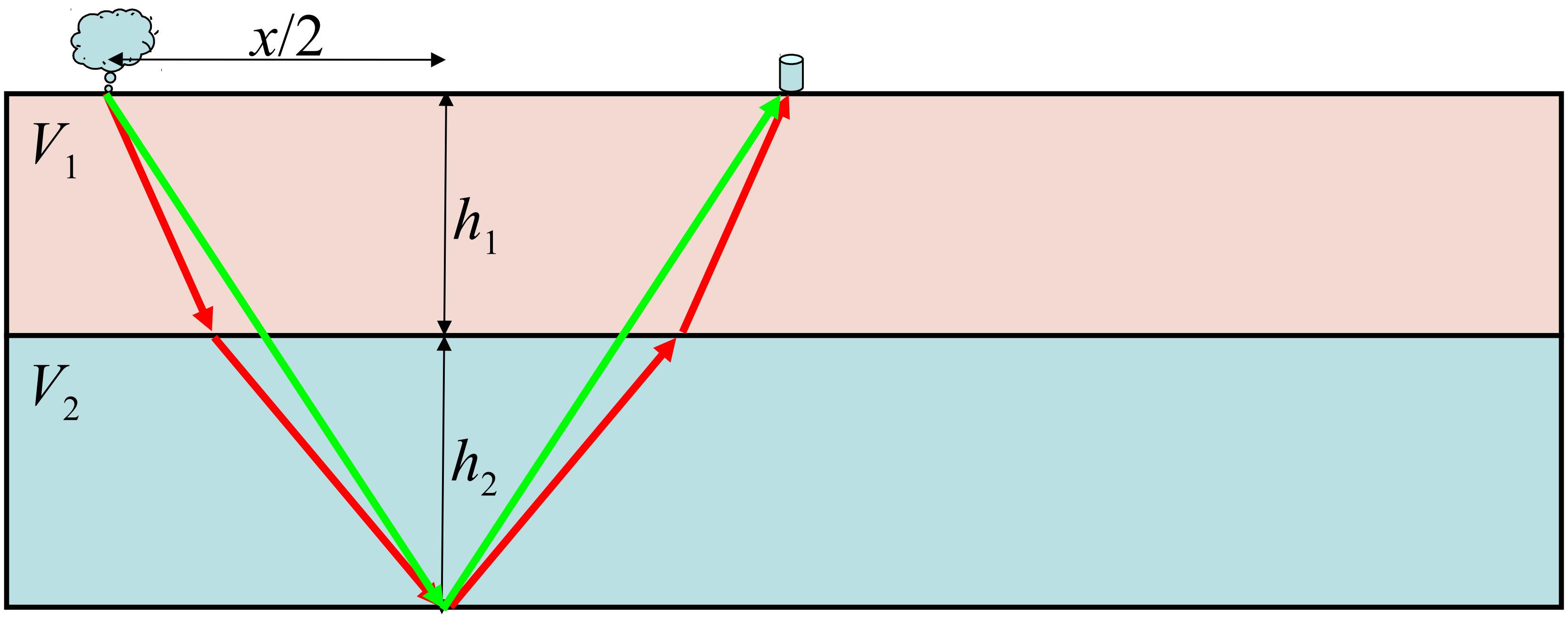
Also works for multiple layers– However instead of explicitly solving e.g.:

$$V_2^2\left(t-\frac{2h_1\sqrt{V_2^2-V_1^2}}{V_1V_2}\right)^2-x^2=\left\{2\left[h_1\left(1-\frac{\sqrt{V_2^2-V_1^1}}{V_1V_2}\right)+h_2\right]\right\}^2$$



We use an approximation called the ***Dix Equation***

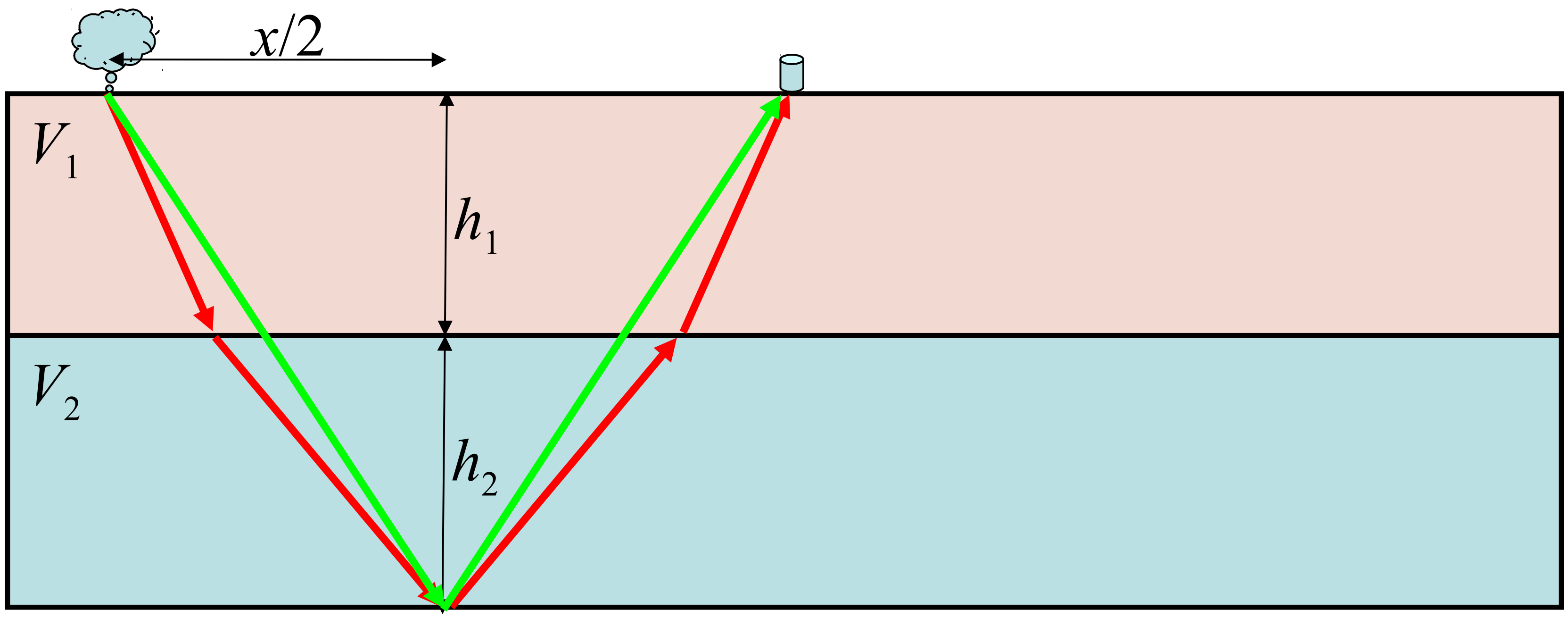
Recall multiple layer case:



Solving for path length in each layer using Snell's law results in a complicated function of all velocities & thicknesses…

Green's method: Uses $x^2 - t^2$ and ignores bending of the ray to get approximate thicknesses and velocities of second, third etc layers using the single-layer equation…

Recall multiple layer case:



Green's method:

$$t = 2\sqrt{\frac{x^2}{4} + \left(h_1 + h_2\right)^2}\left(\frac{h_1}{V_1\left(h_1 + h_2\right)} + \frac{h_2}{V_2\left(h_1 + h_2\right)}\right)$$

$$t^2 = \left[x^2 + 4\left(h_1 + h_2\right)^2\right]\left(\frac{h_1 V_2 + h_2 V_1}{V_1 V_2\left(h_1 + h_2\right)}\right)^2$$

Dix' Equation:

Improves slightly on Green's method because it is a closer approximation of the true physics. We define a ***root mean square (RMS) velocity*** for n layers as:

$$V_{n_{RMS}}^2 \cong \frac{\sum_{i=1}^{n} V_i^2 \Delta t_i}{\sum_{i=1}^{n} \Delta t_i} = \frac{\sum_{i=1}^{n} V_i \, h_i}{\sum_{i=1}^{n} \frac{h_i}{V_i}}$$

Here Δt_i is the one-way vertical travel-time through layer i, i.e.,

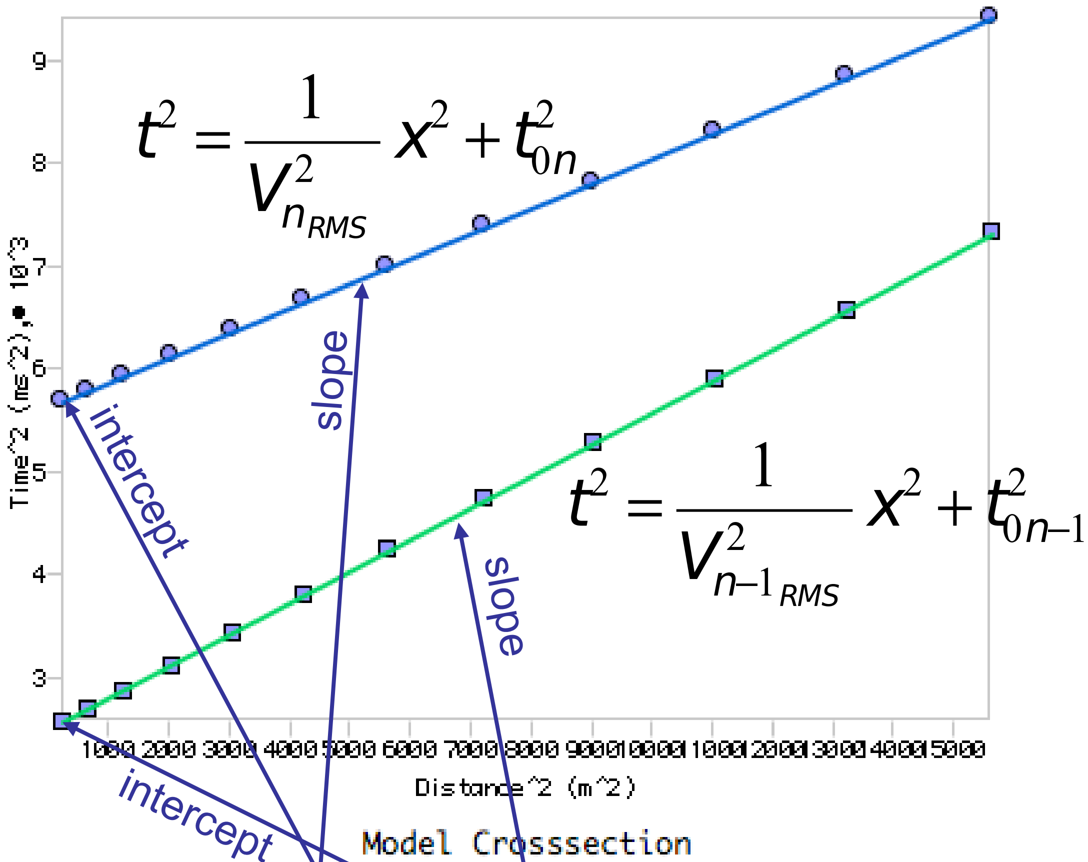
$$\Delta t_i = \frac{h_i}{V_i}$$

Then on an $x^2 - t^2$ plot,

$$t^2 = \frac{1}{V_{n_{RMS}}^2} x^2 + t_0^2 \quad \text{for layer } n$$

$$V_n^2 = \frac{V_{n_{RMS}}^2 \sum_{i=1}^{n} \Delta t_i - V_{n-1_{RMS}}^2 \sum_{i=1}^{n-1} \Delta t_i}{\Delta t_n} \; ; \quad \sum_{i=1}^{n} \Delta t_i = \frac{t_{0n}}{2} \; ; \quad \Delta t_n = \frac{t_{0n} - t_{0n-1}}{2}$$

$$\Rightarrow V_n^2 = \frac{V_{n_{RMS}}^2 t_{0n} - V_{n-1_{RMS}}^2 t_{0n-1}}{t_{0n} - t_{0n-1}}$$



This means we can solve for both the velocity and thickness in any layer for which we have a reflection above & a reflection below, using parameters of a line-fit on an $x^2 - t^2$ plot!

$$\Rightarrow V_n^2 = \frac{V_{n_{RMS}}^2 t_{0n} - V_{n-1_{RMS}}^2 t_{0n-1}}{t_{0n} - t_{0n-1}} \qquad h_n = \Delta t_n V_n = V_n \frac{t_{0n} - t_{0n-1}}{2}$$

Caveat:

Recall that Dix' equation is an ***approximation***.

After refraction in overlying layers, reflections from the 2nd+ interface, e.g. (2-layer case)

$$V_2^2\left(t - \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_1V_2}\right)^2 - x_1^2 = \left\{2\left[h_1\left(1 - \frac{\sqrt{V_2^2 - V_1^1}}{V_1V_2}\right) + h_2\right]\right\}^2$$

contains a $(t^2 - 2at + a^2)$ term. (We ignored this.)

Generally, approximation is good for small offsets, poorer further out.

Parameters	TRUE Model	Green's 30 m	Green's 60 m	Green's 120 m	Dix' 30 m	Dix' 60 m	Dix' 120 m
Velocity 1 (m/s)	400	400	400	400	400	400	400
Velocity 2 (m/s)	1800	2250	2293	2411	1812	1839	1912
Velocity 3 (m/s)	3500	5254	5585	6529	3542	3736	4234
Thickness 1 (m)	10	10	10	10	10	10	10
Thickness 2 (m)	40	50	51	54	40	41	43
Thickness 3 (m)	10	15	16	19	10	11	12

Normal Move-Out and the ***dipping layer problem***:

Recall our definition of ***NMO*** for a single horizontal layer:

$$T_{NMO} = t - t_0 = \frac{\sqrt{x^2 + 4h_1^2}}{V_1} - \frac{2h_1}{V_1}$$

We can write:

$$t = \frac{\sqrt{x^2 + 4h_1^2}}{V_1} = \sqrt{\frac{x^2}{V_1^2} + \frac{4h_1^2}{V_1^2}} = t_0\sqrt{1 + \frac{x^2}{V_1^2 t_0^2}}$$

The ***binomial theorem*** tells us that:

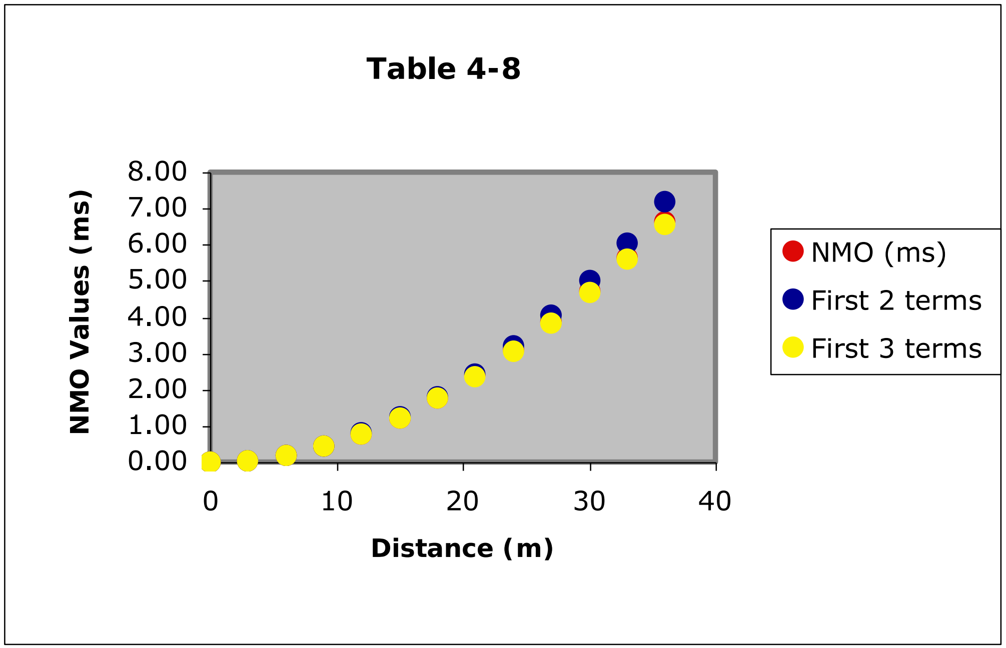
$$(1+z)^a = 1 + az + \frac{a(a-1)}{2!}z^2 + \frac{a(a-1)(a-2)}{3!}z^3 + \cdots$$

So letting $a = \frac{1}{2}$ and $z = \frac{x^2}{V_1^2 t_0^2}$, and ***assuming*** $z << 1$ (i.e. $\frac{x}{2h_1} << 1$) then

$$t \doteq t_0 + \frac{x^2}{2t_0V_1^2} \quad \text{and} \quad T_{NMO} \doteq \frac{x^2}{2t_0V_1^2}$$

Note: If x is not $<< 2h$, we could expand the series further…

$$T_{NMO} \doteq \frac{x^2}{2t_0V_1^2} - \frac{x^4}{8t_0^3V_1^4}$$ (but that gets messy)



This from the text, using a 1500 m/s velocity in a 30 m thick layer…

Why express the reflection travel-time this way if it is inaccurate?

Because now we can write T_{NMO} in terms of the observed t_0!