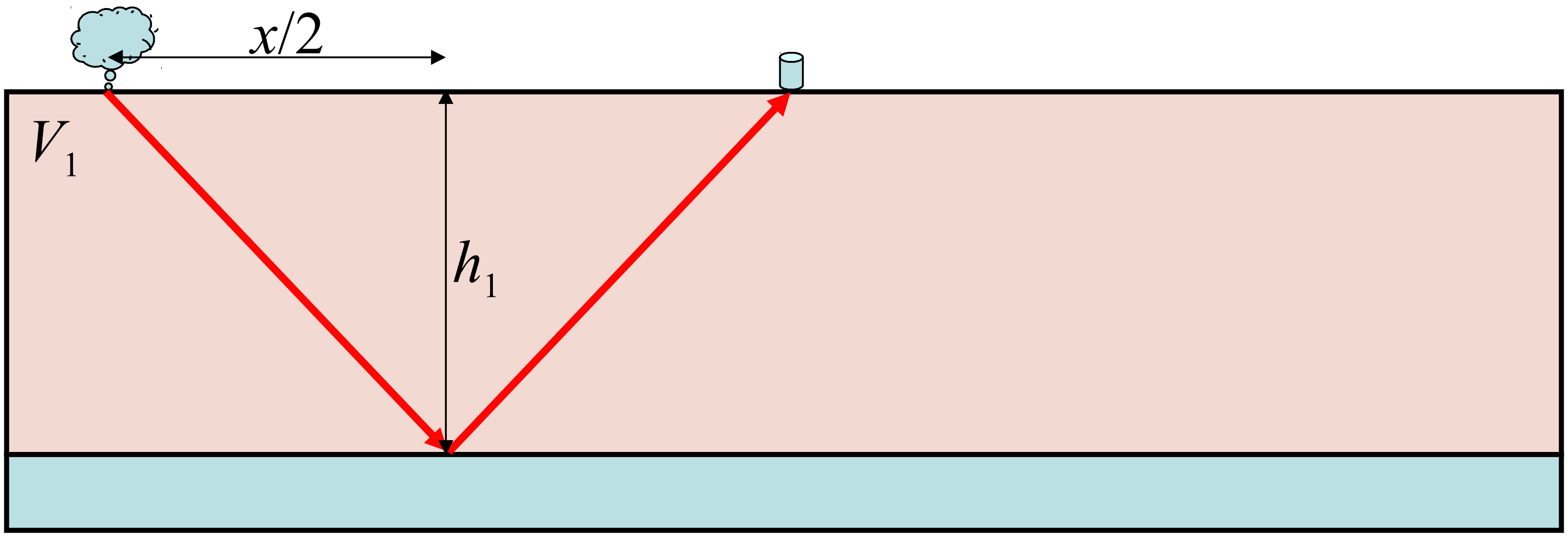
The Reflection Seismic Method:

Consider a single horizontal layer over a half-space, with layer thickness h_1 and velocity V_1:



The travel-time for a reflected wave to a geophone at a distance x from the shot is given by:

$$t = \frac{2\sqrt{\left(\frac{x}{2}\right)^2 + h_1^2}}{V_1} = \frac{\sqrt{x^2 + 4h_1^2}}{V_1}$$

The Reflection Seismic Method:

Hyperbola:

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

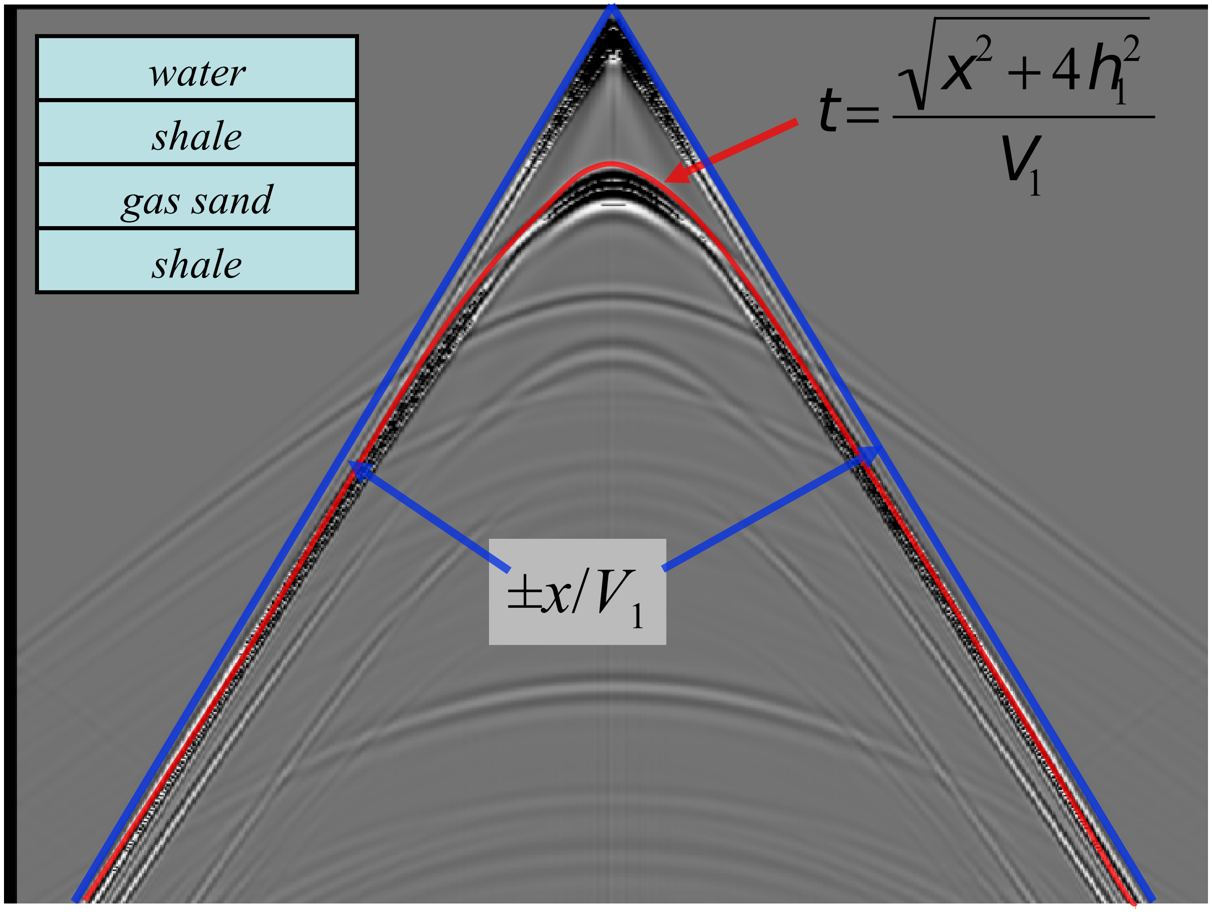
intercept $= b$;
asymptote $m = b/a$

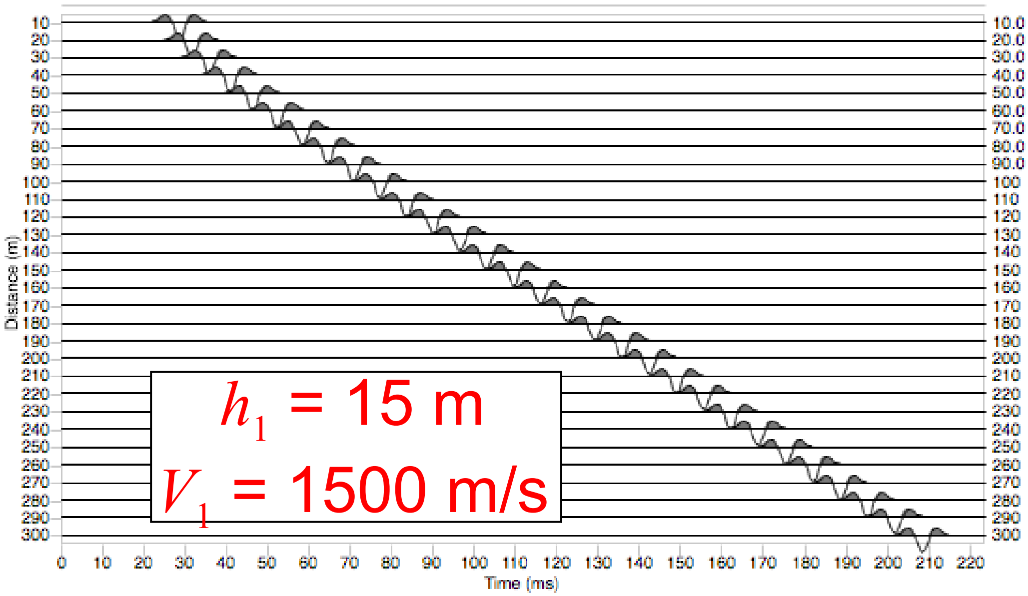
The travel-time for a reflection corresponds to the equation of a ***hyperbola***. If we re-write:

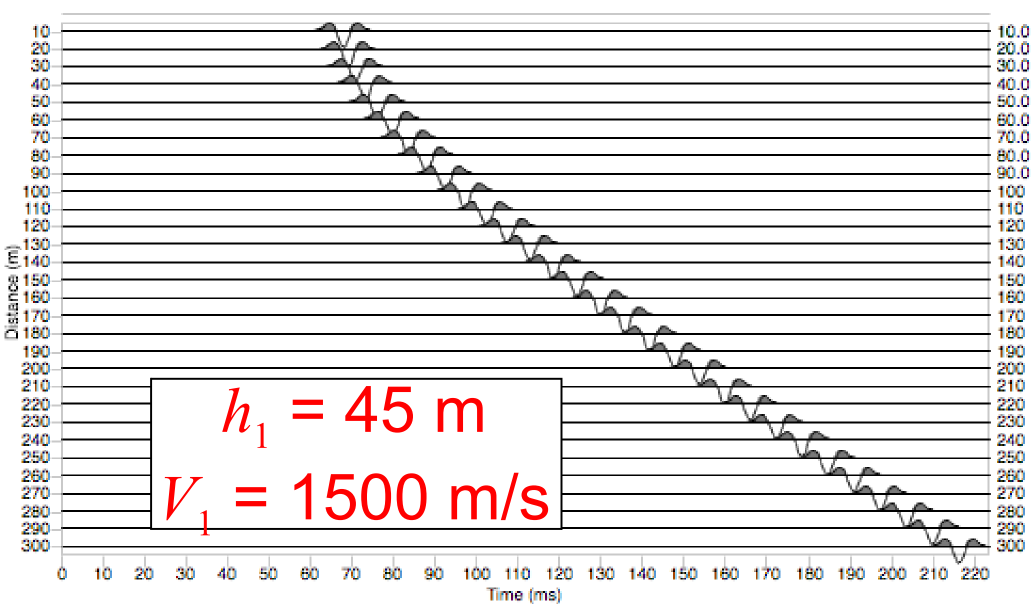
$$t^2 = \frac{x^2}{V_1^2} + \frac{4h_1^2}{V_1^2}$$

$$\Rightarrow \frac{t^2}{\left(2h_1/V_1\right)^2} - \frac{x^2}{\left(2h_1\right)^2} = 1$$

This implies an intercept at $2h_1/V_1$ and asymptotes with slope $\pm 1/V_1$

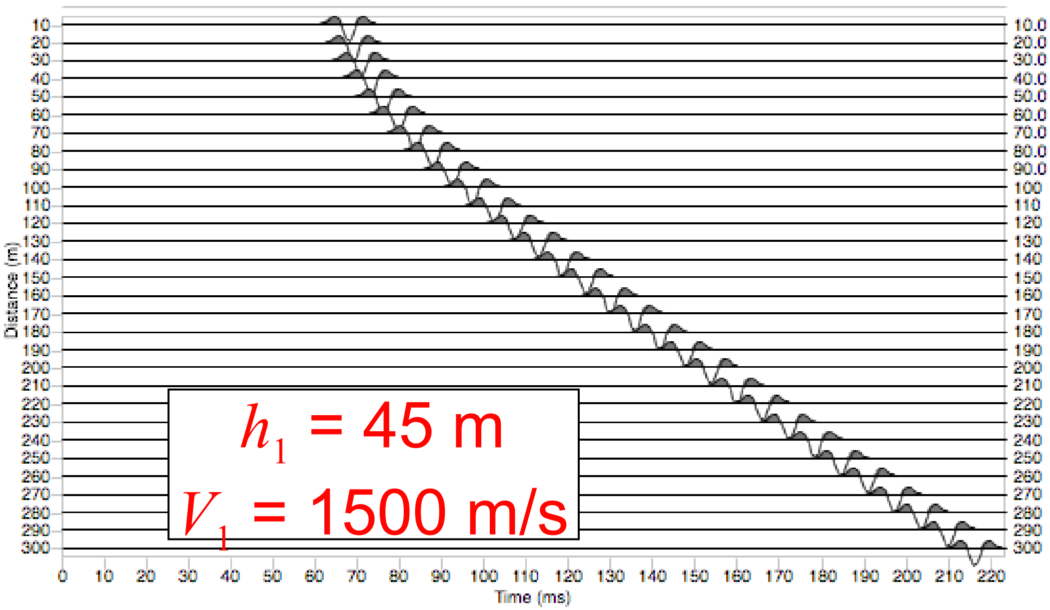


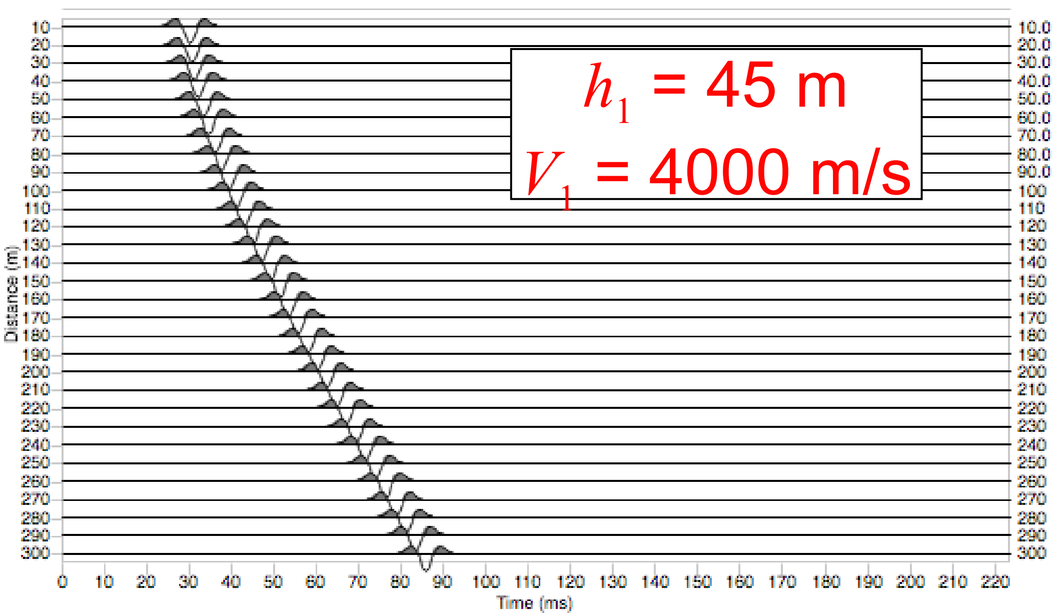




Some quick observations:

Changing only depth of the layer changes intercept of the hyperbola but not the slope or intercept of the asymptotes, so a reflection from a shallower interface appears more “pointy”





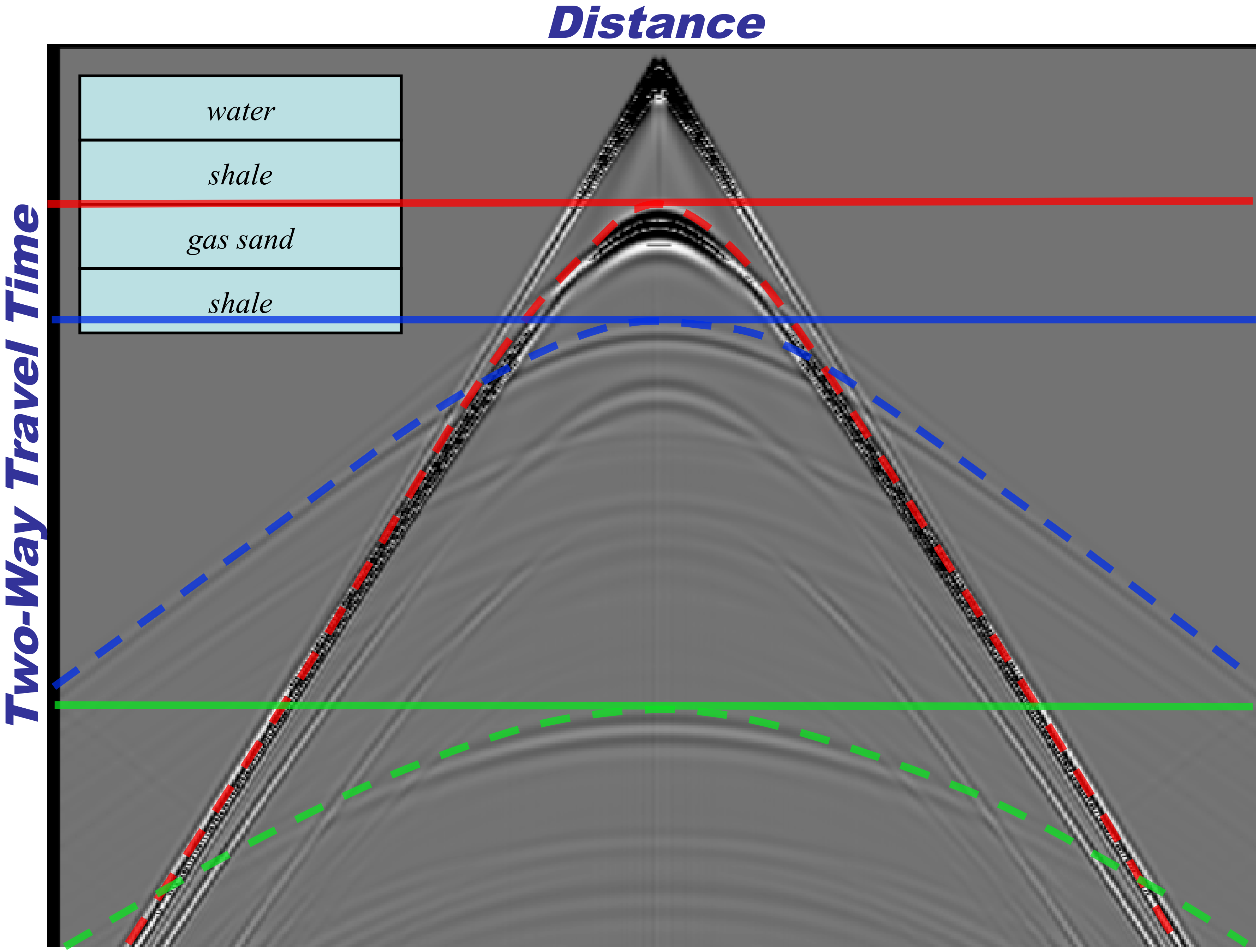
Changing velocity of the layer changes intercept of the hyperbola and the slope of the asymptotes, so a reflection in a layer with higher velocity arrives sooner and appears more "flat"

Normal Move-Out (NMO) is the difference in reflection travel times at distance x relative to the travel time at the intercept ($x = 0$), i.e.,

$$t_{NMO} = \frac{\sqrt{x^2 + 4h_1^2}}{V_1} - \frac{2h_1}{V_1}$$

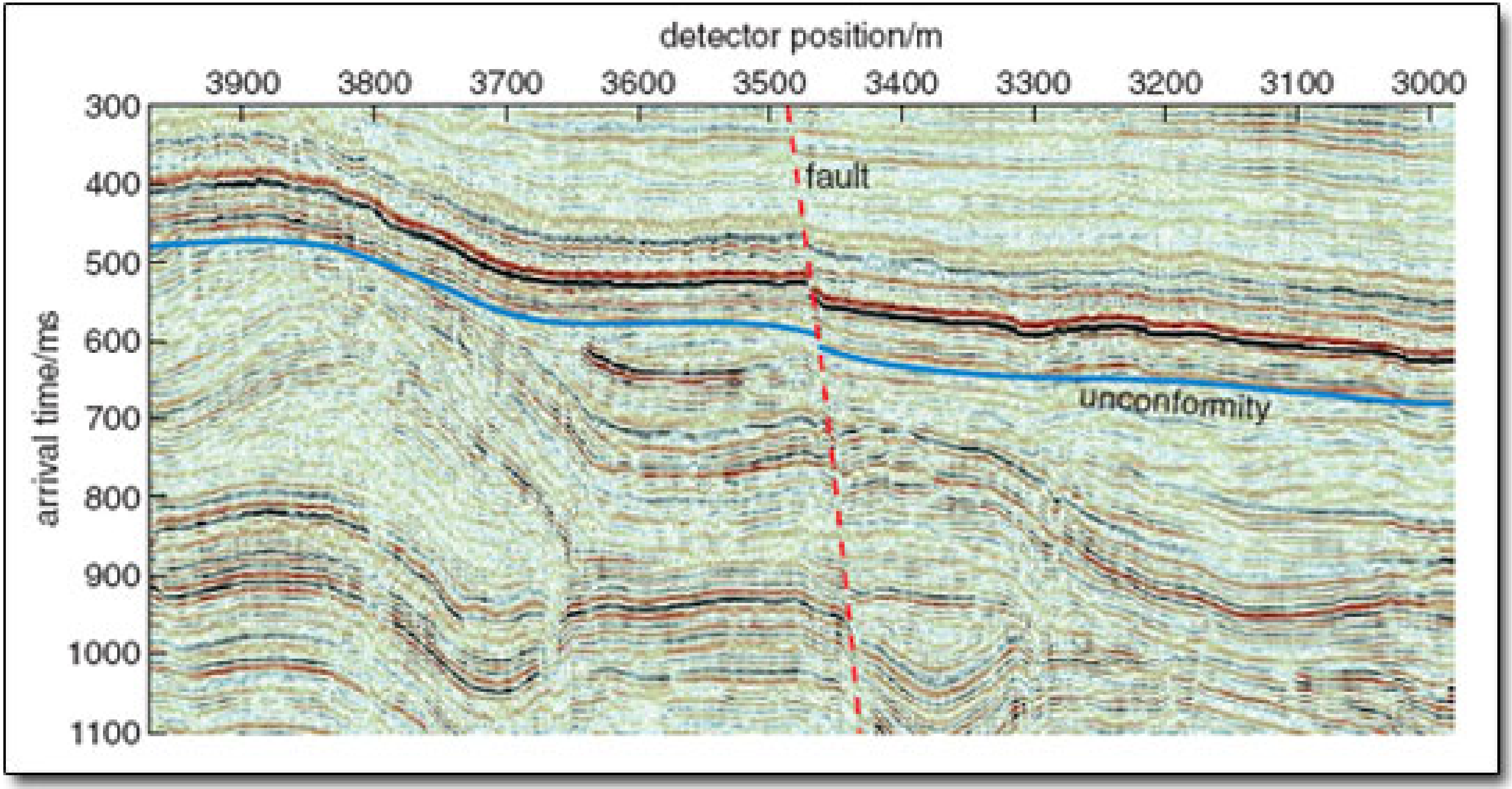
NMO emphasizes changes in curvature of the hyperbola (i.e., it is greater for shallower depth of reflection and for lower velocity of the layer).

The reason we accord special status to NMO is that we will need to ***correct for move-out*** if we want to use the reflection energy to image the subsurface as a ***seismic section***…



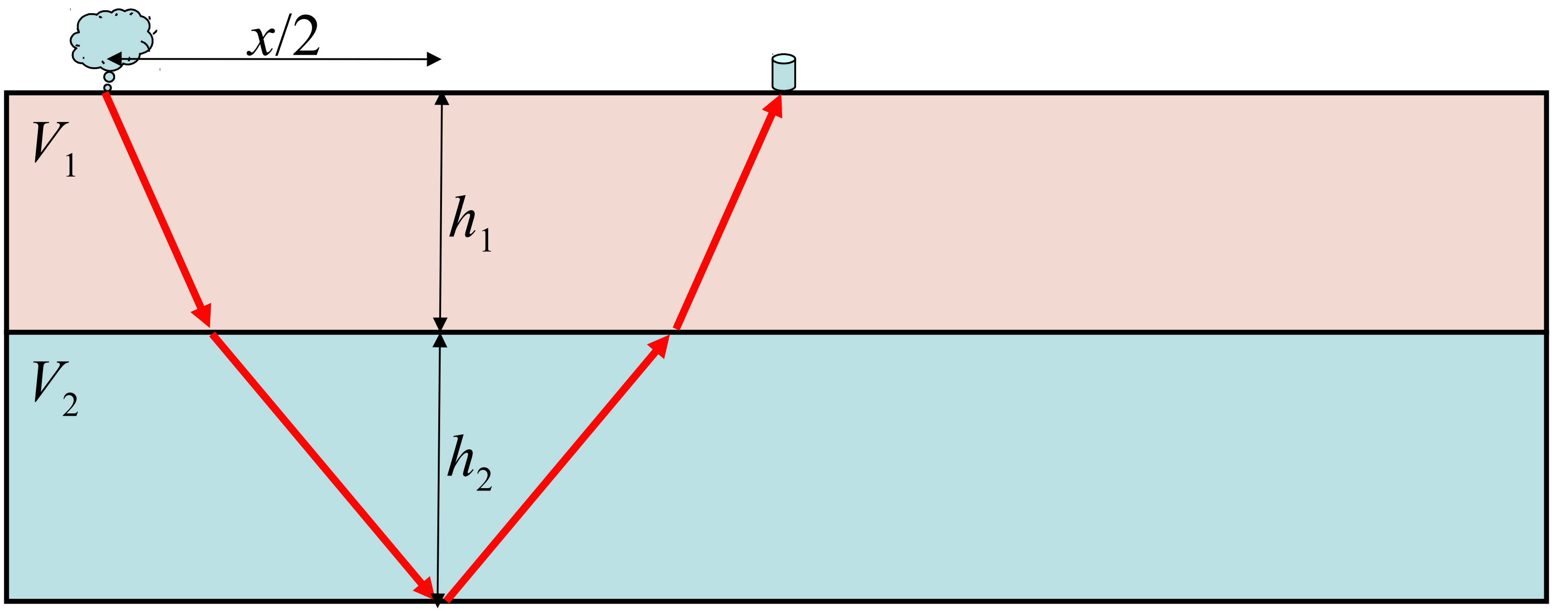
Distance
Two-Way Travel Time
water
shale
gas sand
shale

Shifting the seismic reflection amplitude to where it would be (in two-way travel-time) for zero-offset produces an image…



Note that this approach is ***VERY*** different from the model of velocity structure we generate from the refraction method!

Reflection from a second layer interface over half-space:



Can derive using Snell's law but easier to consider that:

- For $x = 0$, $t = 2\left(\frac{h_1}{V_1} + \frac{h_2}{V_2}\right)$ $\Rightarrow$ ***intercept of hyperbola***
- For $x \to \infty$, ***asymptotic to the layer 2 refraction***

$$t = \frac{x}{V_2} + \frac{2h_1\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

Equation for the hyperbola then is

$$\frac{\left(t-t_0\right)^2}{a^2}-\frac{x^2}{b^2}=1 \Rightarrow a=2\left(\frac{h_1}{V_1}+\frac{h_2}{V_2}\right); \frac{b}{a}=\frac{1}{V_2}; t_0=\frac{2h_1\sqrt{V_2^2-V_1^2}}{V_1V_2}$$

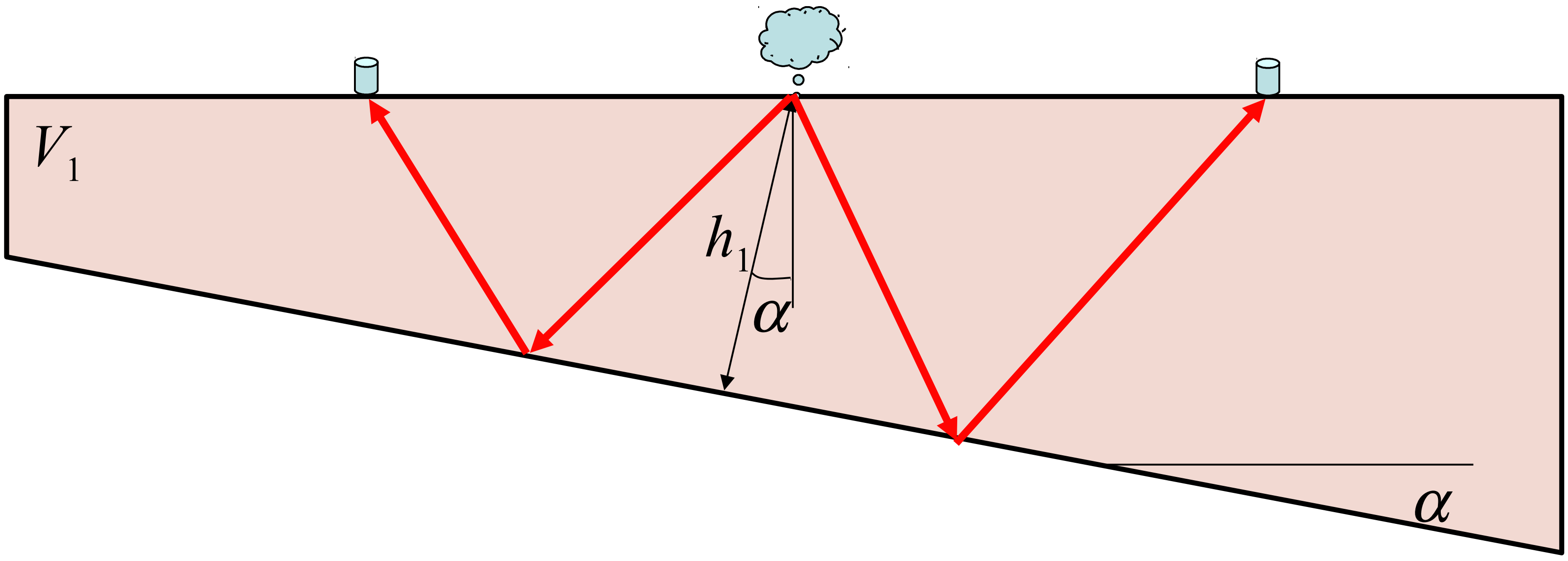
After some algebra we have

$$V_2^2\left(t-\frac{2h_1\sqrt{V_2^2-V_1^2}}{V_1V_2}\right)^2-x_1^2=\left\{2\left[h_1\left(1-\frac{\sqrt{V_2^2-V_1^2}}{V_1V_2}\right)+h_2\right]\right\}^2$$

And you might see where this could start to get complicated for 3, 4, … layers

This is part of why industry seismic reflection processing historically did not go after full seismic velocity analysis but instead took shortcuts to imaging of structures…

A Dipping Reflector:



Geometrically, this is equivalent to rotating the axis of the reflector by the dip angle α. This rotates the hyperbola on the travel-time curve by $\theta = \tan^{-1}(-2h_1 \sin\alpha)$ and has equation

$$V_1^2 t^2 - x^2 = 4h_1\left(h_1 - x\sin\alpha\right)$$