Summary: Refraction Method, Horizontal Layers

- For a ***single horizontal layer over a halfspace***, observed travel-times for ***direct*** and ***refracted*** arrivals:

$$t = \frac{x}{V_1} \qquad\qquad t = \frac{x}{V_2} + \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

give the velocities and layer thickness from $t = m_i x + b_i$ as:

$$V_1 = \frac{1}{m_1} \qquad\qquad V_2 = \frac{1}{m_2} \qquad\qquad h = \frac{b_2 V_1 V_2}{2\sqrt{V_2^2 - V_1^2}}$$

- The ***crossover distance***, x_{co} can be used in experiment design, to ensure adequate geophone sampling:

$$x_{co} = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}}$$

- For an n-layer Earth:

$$t = \frac{x}{V_n} + \sum_{i=1}^{n-1} \frac{2h_i\sqrt{V_n^2 - V_i^2}}{V_n V_i}$$

Summary: Refraction Method, Dipping Layers

Dipping Layer t-x equations: Graphical solution

$$\sin(\theta_c) = \frac{V_1}{V_2}$$

We have:

$$t_D = \frac{x}{V_1} \qquad \Rightarrow \qquad m_0 = 1/V_1$$

$$t_u = \frac{x}{V_1}\sin(\theta_c - \alpha) + \frac{2h_u\cos(\theta_c)}{V_1} = m_u x + t_{0u}$$

$$t_d = \frac{x}{V_1}\sin(\theta_c + \alpha) + \frac{2h_d\cos(\theta_c)}{V_1} = m_d x + t_{0d}$$

(1) $$V_1 = \frac{1}{m_0} = \frac{x}{t}(x \le x_{co})$$

(2) $$\alpha = \frac{\sin^{-1}\left(\frac{m_d}{m_0}\right) - \sin^{-1}\left(\frac{m_u}{m_0}\right)}{2}$$

(3) $$\theta_c = \frac{\sin^{-1}\left(\frac{m_d}{m_0}\right) + \sin^{-1}\left(\frac{m_u}{m_0}\right)}{2}$$

and

(4)

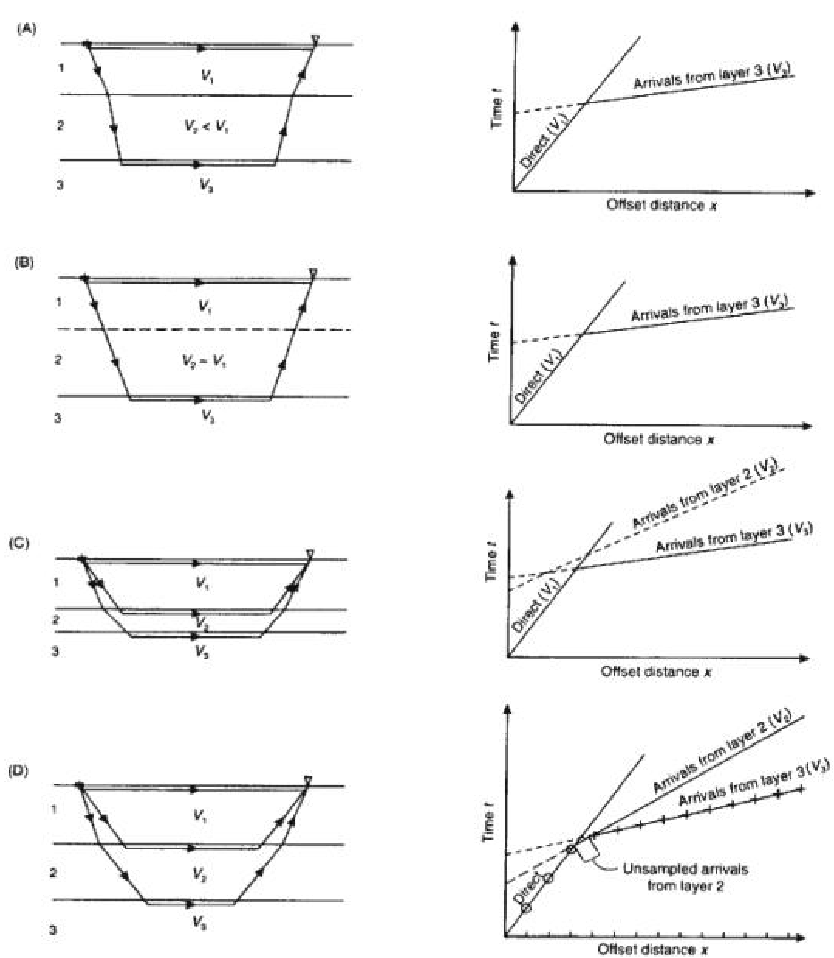
$$V_2 = \frac{V_1}{\sin(\theta_c)} \qquad (a)$$

$$h_d = \frac{t_{0d}}{2m_0\cos(\theta_c)} \qquad (b)$$

$$h_u = \frac{t_{0u}}{2m_0\cos(\theta_c)} \qquad (c)$$

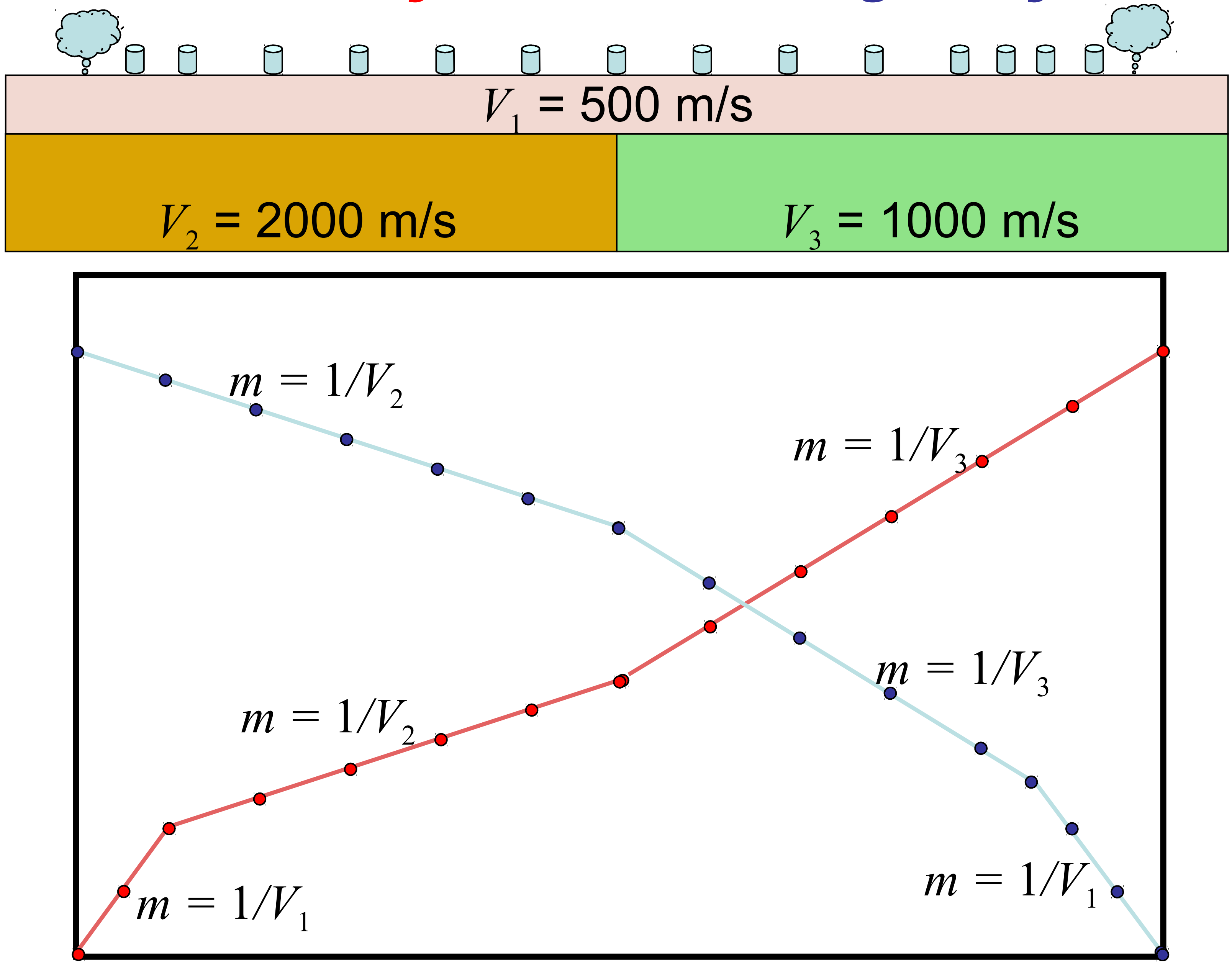
Summary: Hidden Layer Problem

A. Velocity inversion produces no critical refraction from layer 2
B. Insufficient velocity contrast makes refraction difficult to identify
C. Refraction from thin layer does not become first arrival
D. Geophone spacing too large to identify second refraction

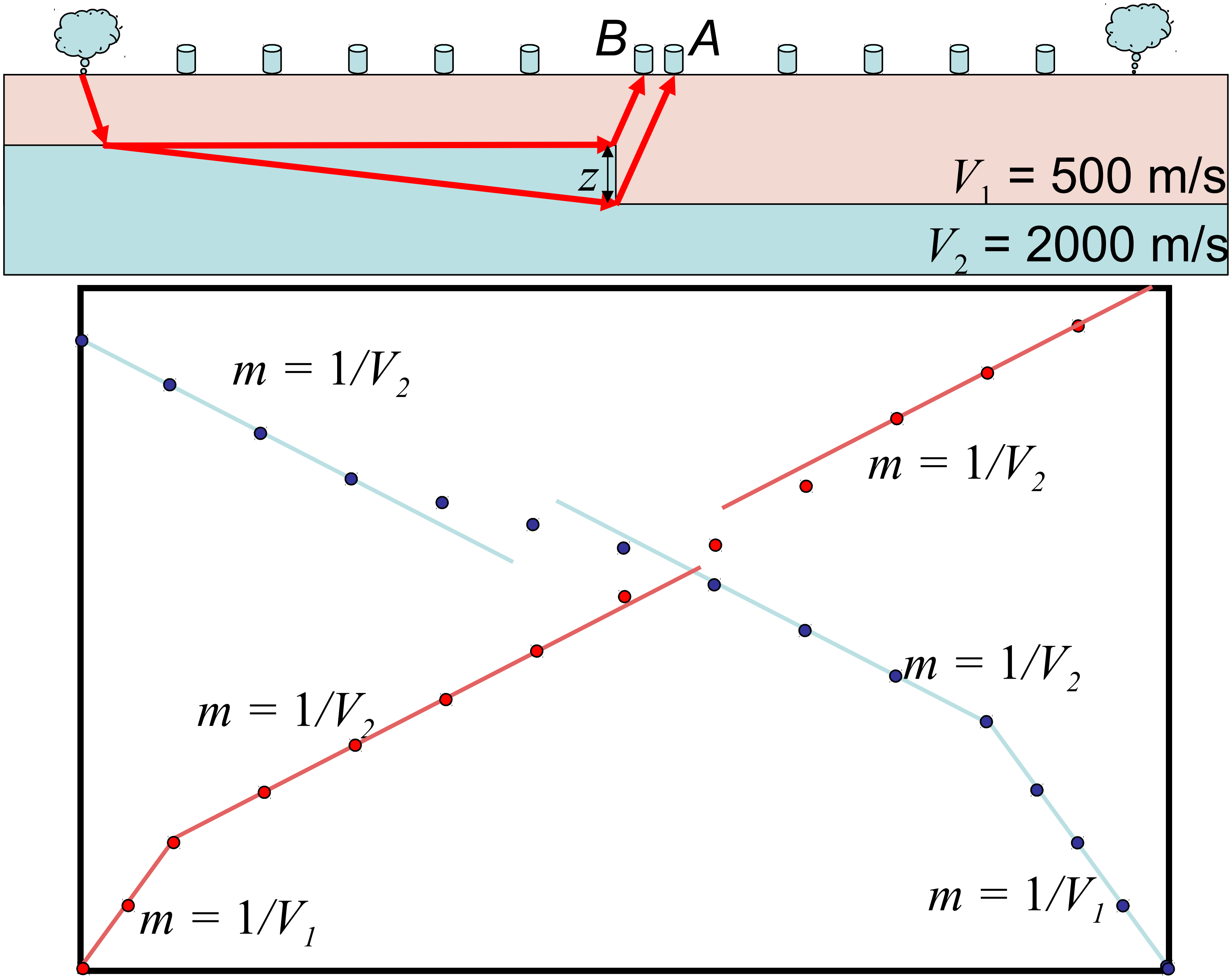


http://www.ukm.my/rahim/Seismic%20Refraction%20Surveying.htm

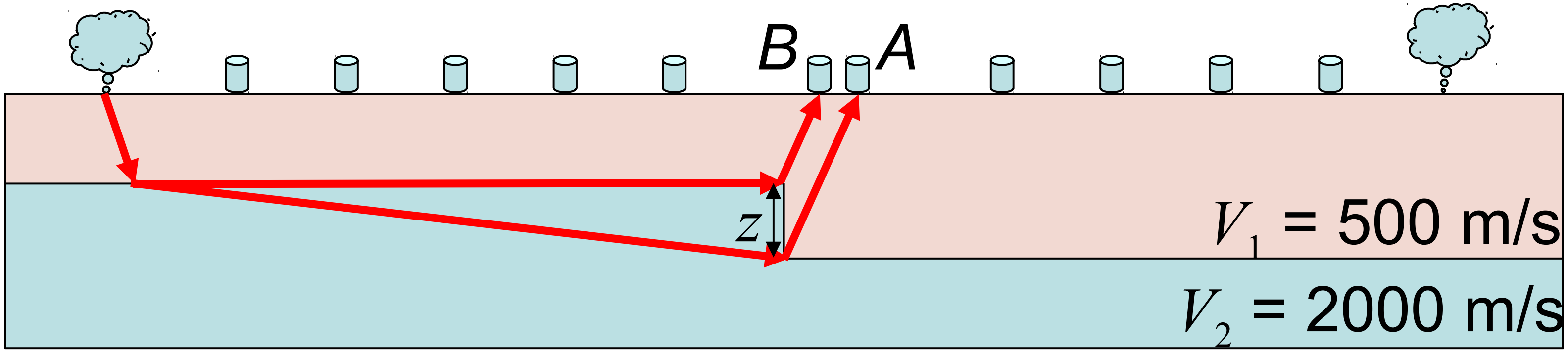
Summary: Lateral Heterogeneity



Summary: Vertical Offset



Summary: *Vertical Offset*

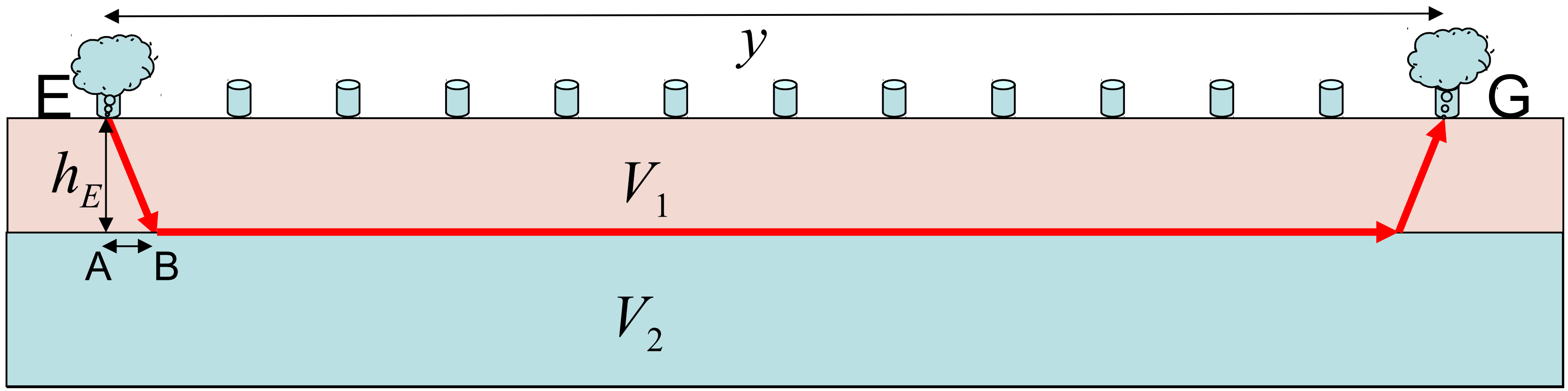


Difference in the intercept times between geophones A & B, is directly proportional to the path difference between the returning head-wave rays:

$$t_{0a} - t_{0b} = \frac{z\cos i_c}{V_1} = \frac{z\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

$$z = \frac{\left(t_{0a} - t_{0b}\right) V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$

Refraction from an irregular surface: ***Delay-Time Method***



Define ***delay time*** as the time the ray traveled in layer 1 along a "slant path", less the time it would have taken to travel the horizontal distance (AB) at velocity V_2. Thus, the total delay time τ_{EG} traveling from E to G (or G to E) is

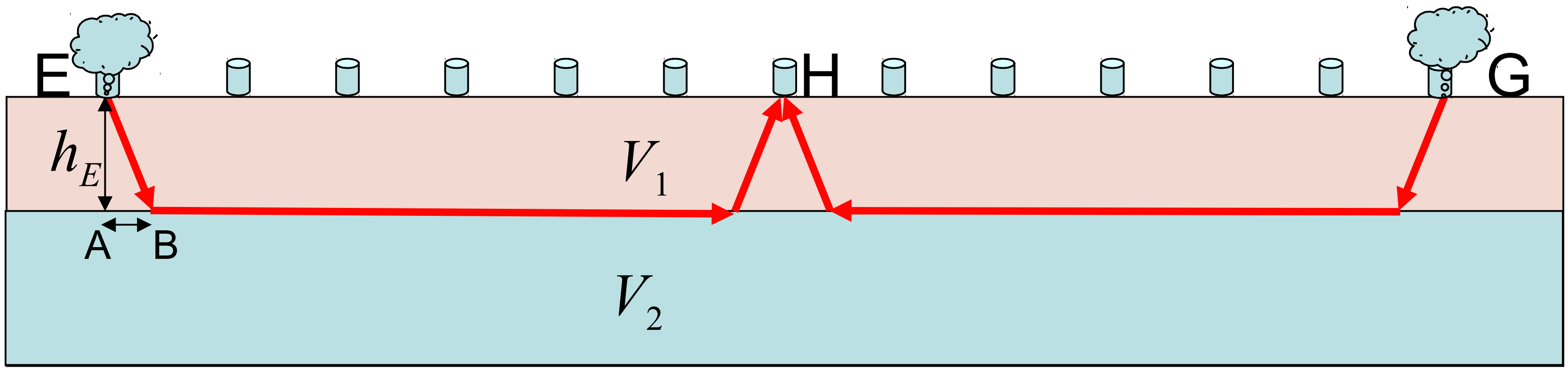
$$\tau_{EG} = t_R - \frac{y}{V_2}$$, where t_R is total travel time.

Delay time under E: $$\tau_E = t_R - \frac{y}{V_2} - \tau_G = \frac{EB}{V_1} - \frac{AB}{V_2}$$

$$\tau_E = \frac{h_E}{V_1 \cos i_c} - \frac{h_E \tan i_c}{V_2} = h_E \frac{\sqrt{V_2^2 - V_1^2}}{V_1 V_2} \Rightarrow h_E = \tau_E \frac{V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$

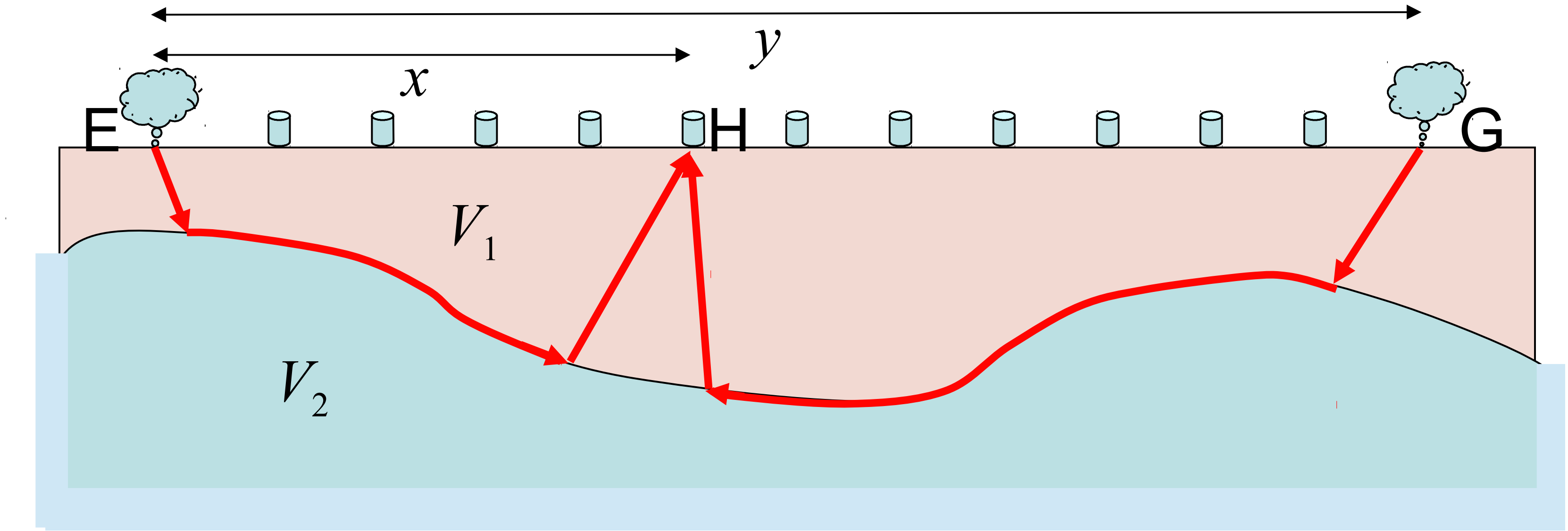
(This is half of the "time intercept" on our t–x plots!)

We can't measure it directly, but with reversed shots:



τ_H from E ≈ τ_H from G, and $\tau_H = \dfrac{t_{EH} + t_{GH} - t_{EG}}{2}$

Refraction from an irregular surface: ***Delay-Time Method***



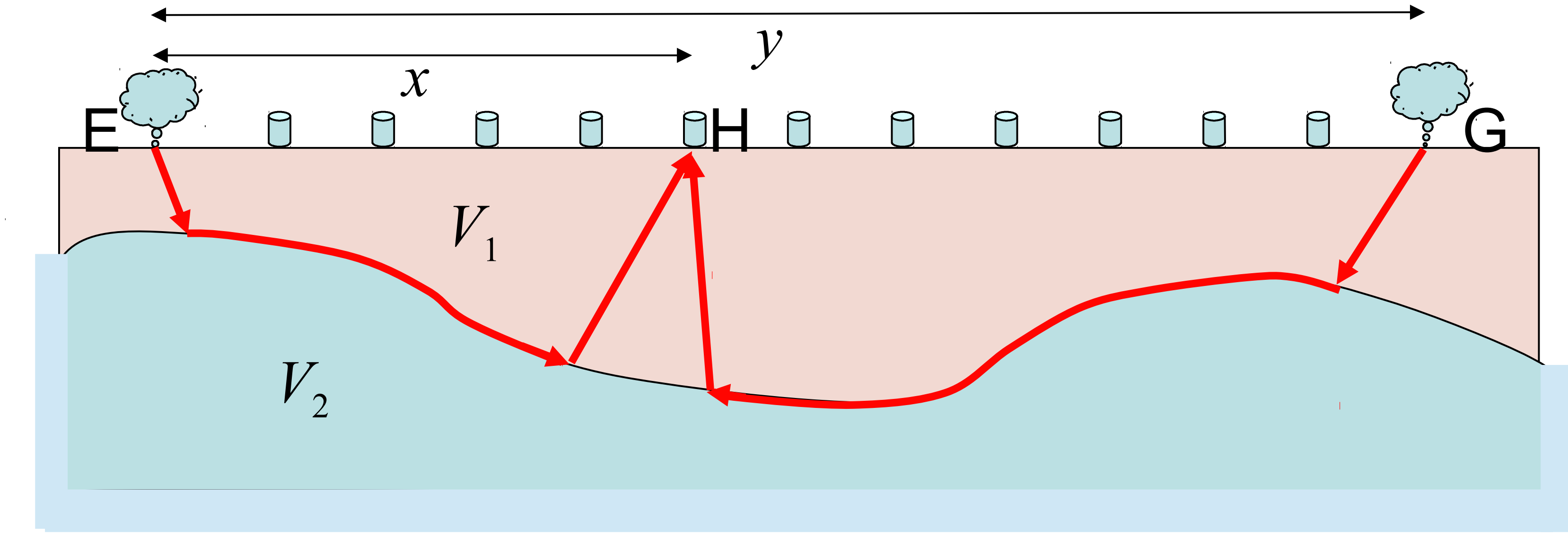
$$\tau_H = \frac{t_{EH} + t_{GH} - t_{EG}}{2} \qquad h_H = \tau_H \frac{V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$

Problem however: to get h_H from τ_H, we need to know V_2! However, we have:

$$t_{EH} - t_{GH} = \left(\tau_E + \tau_H + \frac{x}{V_2}\right) - \left(\tau_G + \tau_H + \frac{y-x}{V_2}\right) = \tau_E - \tau_G - \frac{y}{V_2} + \frac{2x}{V_2}$$

(⇒ a line with slope $2/V_2$!)

Sometimes also called the ***"plus-minus method"***:



- Plot t_{1i}–t_{2i} vs x_i for SP1, SP2 and all geophones i.
- Calculate V_2 from slope of the line fit ($V_2 = 2/m$).
- At each geophone i, calculate thickness as:

$$h_i = \left(\frac{t_{1i} + t_{2i} - t_{12}}{2} \right) \frac{V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$