Last time: The Refraction Method

- For a ***single horizontal layer over a halfspace***, observed travel-times for ***direct*** and ***refracted*** arrivals:

$$t = \frac{x}{V_1} \qquad\qquad t = \frac{x}{V_2} + \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

give the velocities and layer thickness from $t = m_i x + b_i$ as:

$$V_1 = \frac{1}{m_1} \qquad\qquad V_2 = \frac{1}{m_2} \qquad\qquad h = \frac{b_2 V_1 V_2}{2\sqrt{V_2^2 - V_1^2}}$$

- The ***crossover distance***, x_{co} can be used in experiment design, to ensure adequate geophone sampling:

$$x_{co} = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}}$$

- For an n-layer Earth:

$$t = \frac{x}{V_n} + \sum_{i=1}^{n-1} \frac{2h_i\sqrt{V_n^2 - V_i^2}}{V_n V_i}$$

Read for Monday: ***Burger*** 106-141 (§3.7-3.11)

Last time: The Refraction Method

Like before, we have the ***observed*** travel-times and want to know the ***model parameters***. We know V_1 from the direct arrival, & we know the slopes are:

$$m_u = \frac{\sin(i_c - \alpha)}{V_1} \qquad m_d = \frac{\sin(i_c + \alpha)}{V_1}$$

then we can solve for velocity V_2:

$$i_c = \frac{\sin^{-1}(V_1 m_u) + \sin^{-1}(V_1 m_d)}{2} \left\{ = \sin^{-1}\left(\frac{V_1}{V_2}\right) \right\} \Rightarrow \quad V_2 = V_1 \Big/ \sin\left\{ \frac{\sin^{-1}(V_1 m_u) + \sin^{-1}(V_1 m_d)}{2} \right\}$$

and angle:

$$\alpha = \frac{\sin^{-1}(V_1 m_d) - \sin^{-1}(V_1 m_u)}{2}$$

The normal thicknesses use intercept times t_{0u}, t_{0d}:

$$h_d = \frac{t_{0d} V_1}{2\cos i_c} \qquad h_u = \frac{t_{0u} V_1}{2\cos i_c}$$

(Can get vertical distance from shot to top of layer by dividing h_d or h_u by $\cos\,\alpha$).

Dipping Layer t-x equations: Graphical solution

$$\sin(\theta_c)=\frac{V_1}{V_2}$$

We have:

$$t_D=\frac{x}{V_1} \qquad \Rightarrow \qquad m_0=1/V_1$$

$$t_u=\frac{x}{V_1}\sin(\theta_c-\alpha)+\frac{2h_u\cos(\theta_c)}{V_1}=m_u x+t_{0u}$$

$$t_d=\frac{x}{V_1}\sin(\theta_c+\alpha)+\frac{2h_d\cos(\theta_c)}{V_1}=m_d x+t_{0d}$$

(1) $$V_1=\frac{1}{m_0}=\frac{x}{t}(x\leq x_{co})$$

(2) $$\alpha=\frac{\sin^{-1}\left(\frac{m_d}{m_0}\right)-\sin^{-1}\left(\frac{m_u}{m_0}\right)}{2}$$

(3) $$\theta_c=\frac{\sin^{-1}\left(\frac{m_d}{m_0}\right)+\sin^{-1}\left(\frac{m_u}{m_0}\right)}{2}$$

and

(4)

$$V_2=\frac{V_1}{\sin(\theta_c)} \qquad (a)$$

$$h_d=\frac{t_{0d}}{2m_0\cos(\theta_c)} \qquad (b)$$

$$h_u=\frac{t_{0u}}{2m_0\cos(\theta_c)} \qquad (c)$$

Some Limitations of the Refraction Method:

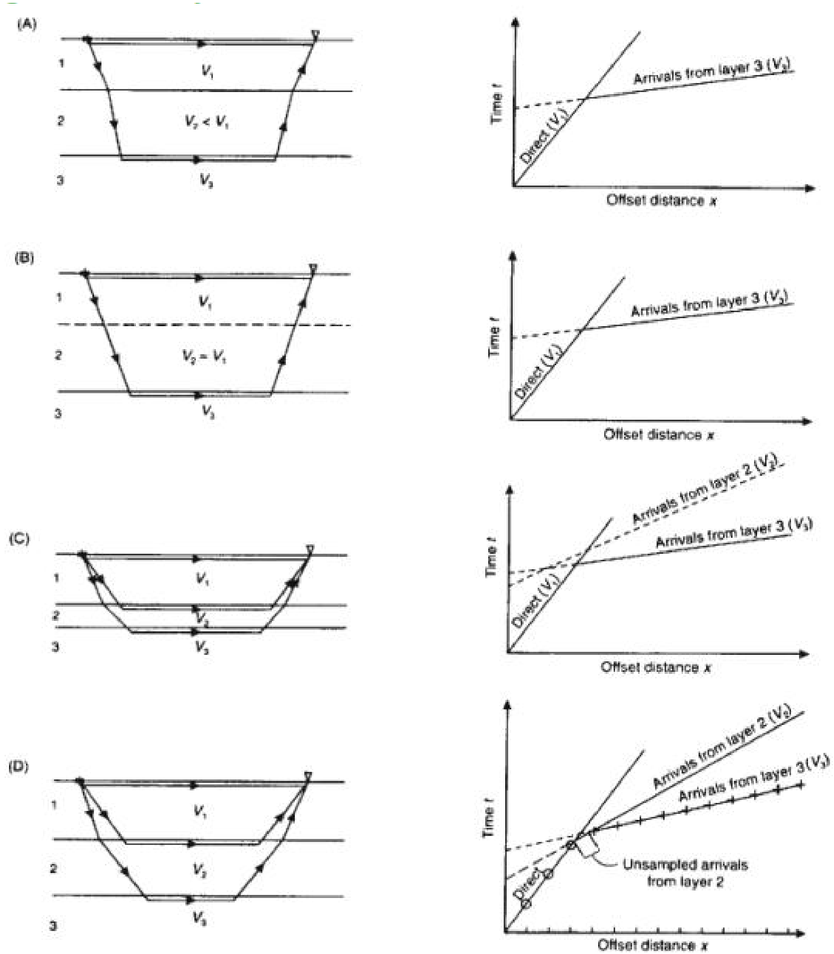
- *It does not get returns from **low velocity layers**.* This means that not only does your best-fitting model not include the low velocity layer, but also estimates of the ***depth*** to top of all subsequent layers will be ***overestimated***.

- ***Thin layers** may be **aliased** by geophone spacing, & in some cases will be missed regardless of spacing*! As with the low velocity layer, this won't affect velocity estimates for lower layers but will affect depth (in this case ***depth is underestimated***).

Hidden Layer Problem

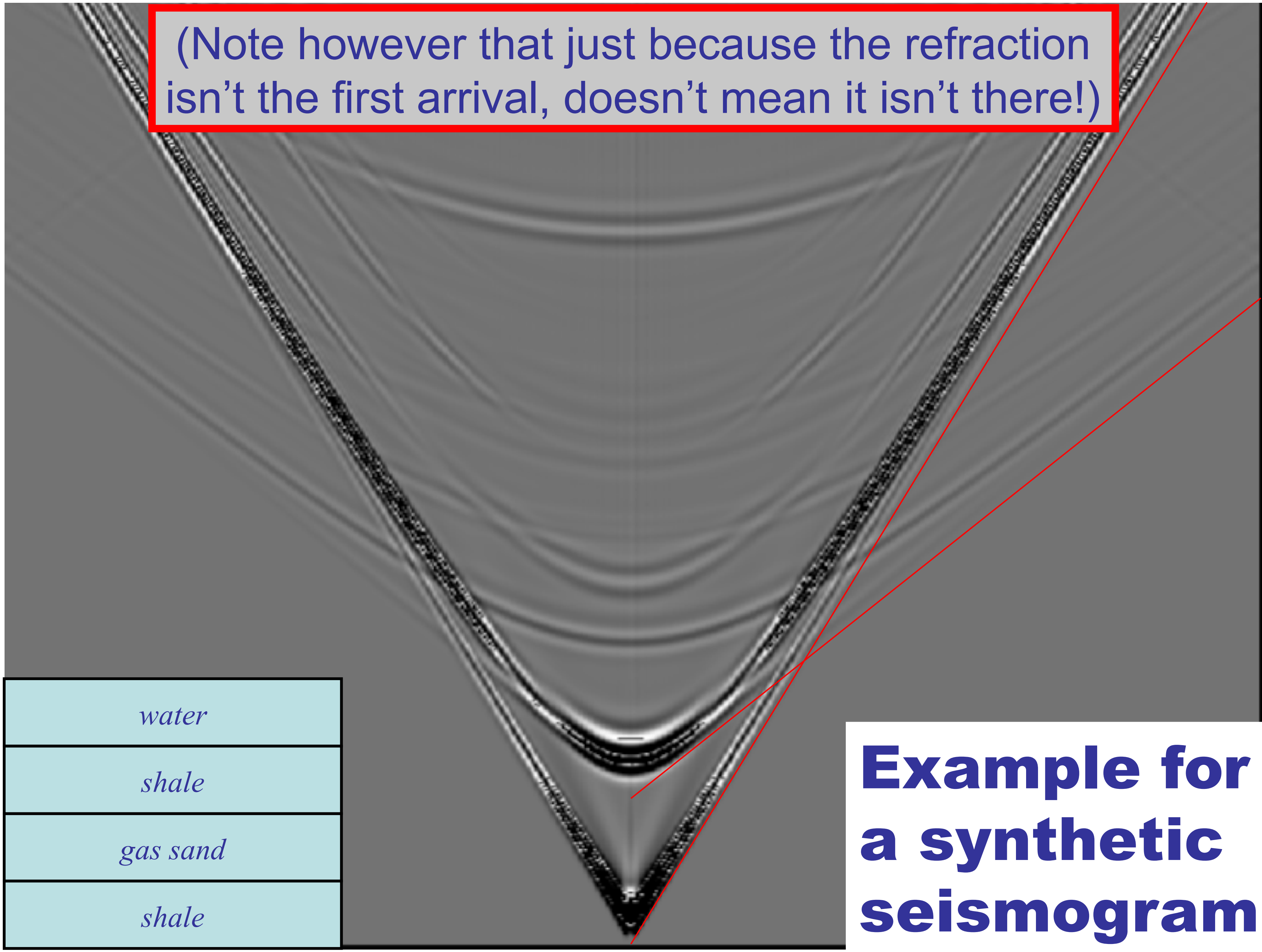
Layers may not be detected by first arrival analysis:

A. Velocity inversion produces no critical refraction from layer 2
B. Insufficient velocity contrast makes refraction difficult to identify
C. Refraction from thin layer does not become first arrival
D. Geophone spacing too large to identify second refraction



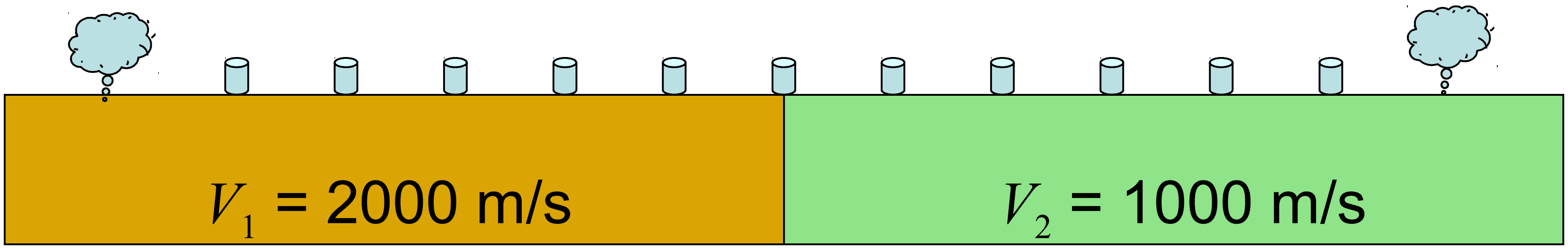
http://www.ukm.my/rahim/Seismic%20Refraction%20Surveying.htm

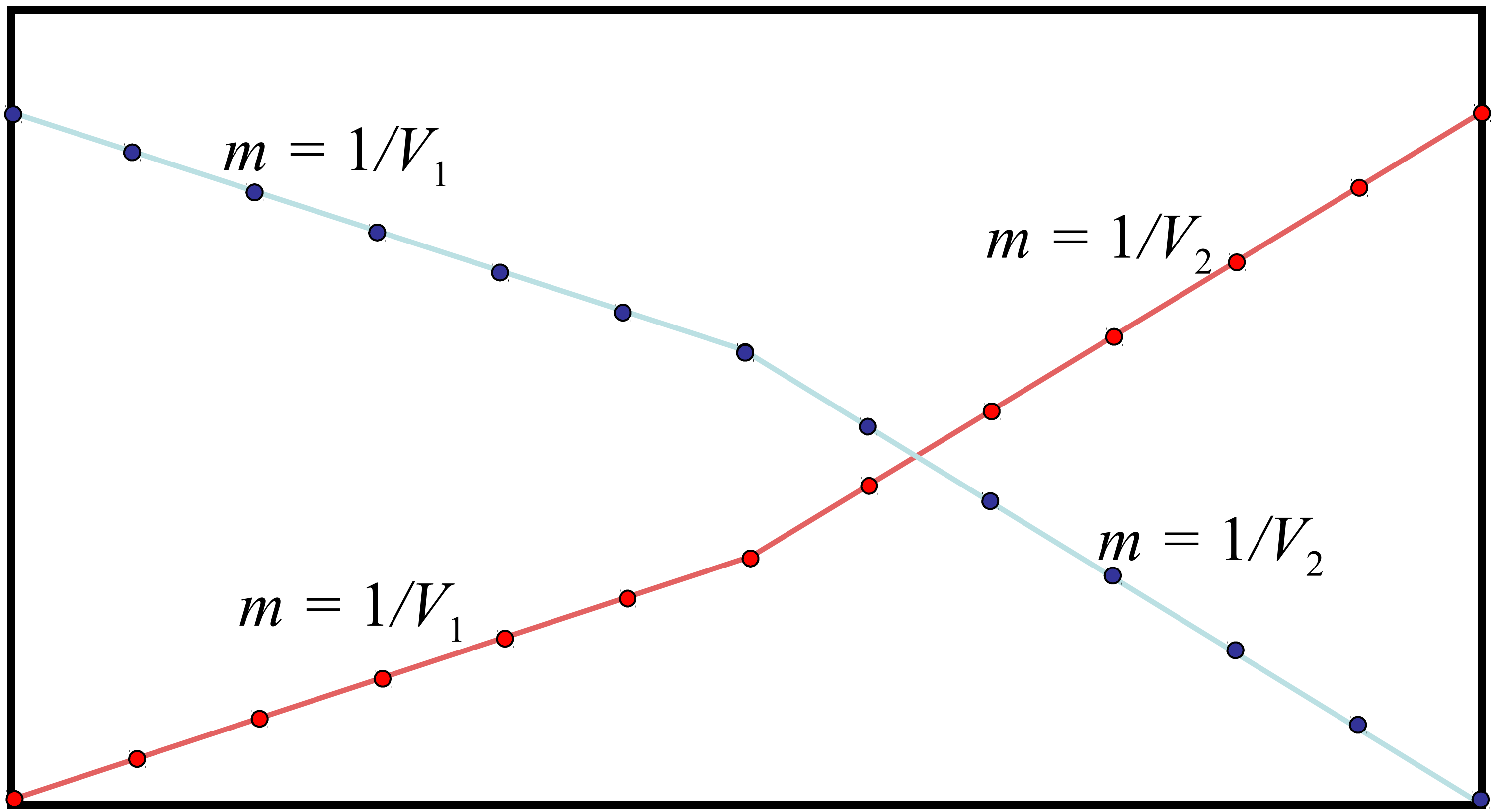
(Note however that just because the refraction isn't the first arrival, doesn't mean it isn't there!)

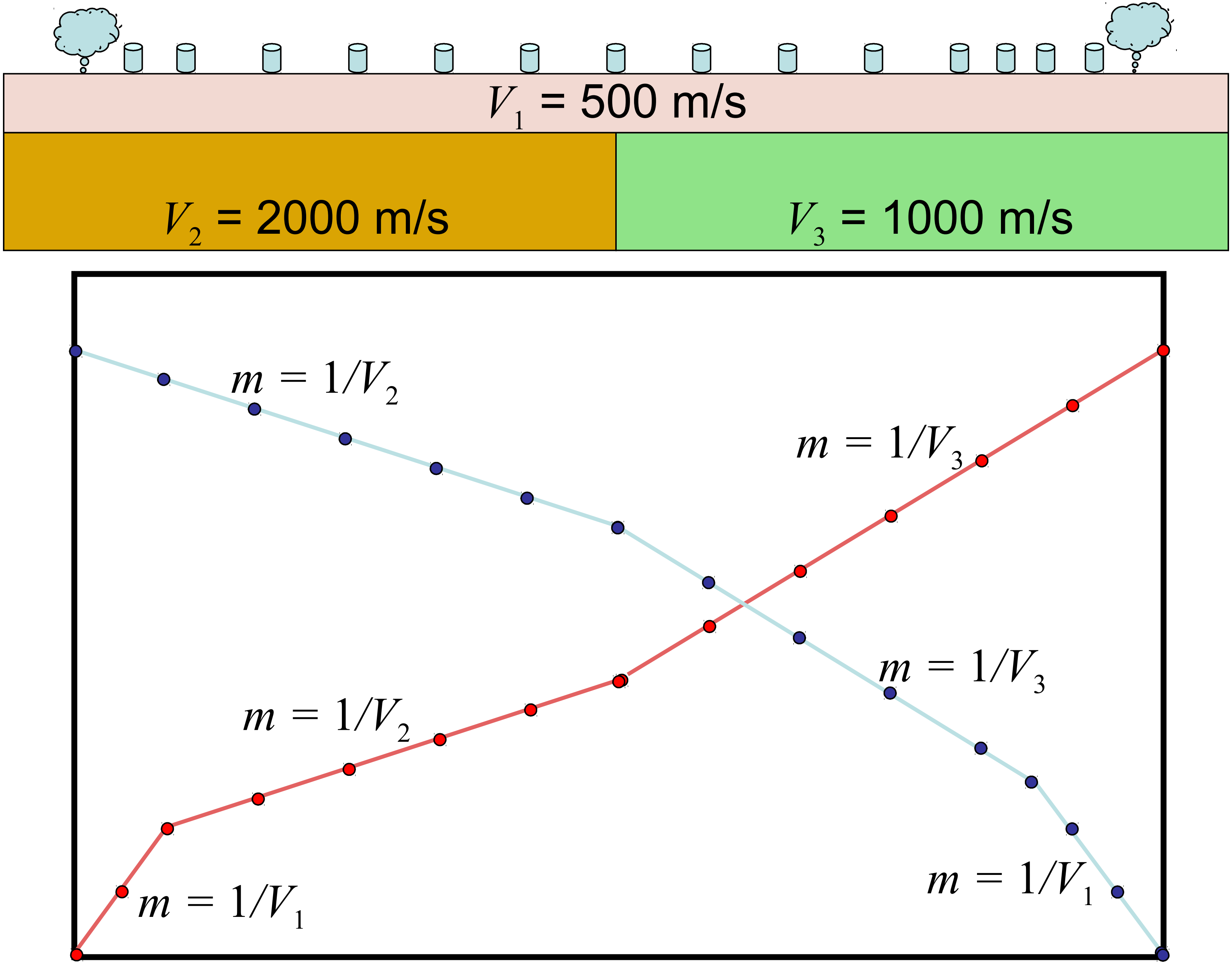


Example for a synthetic seismogram

What if velocity changes within layers?

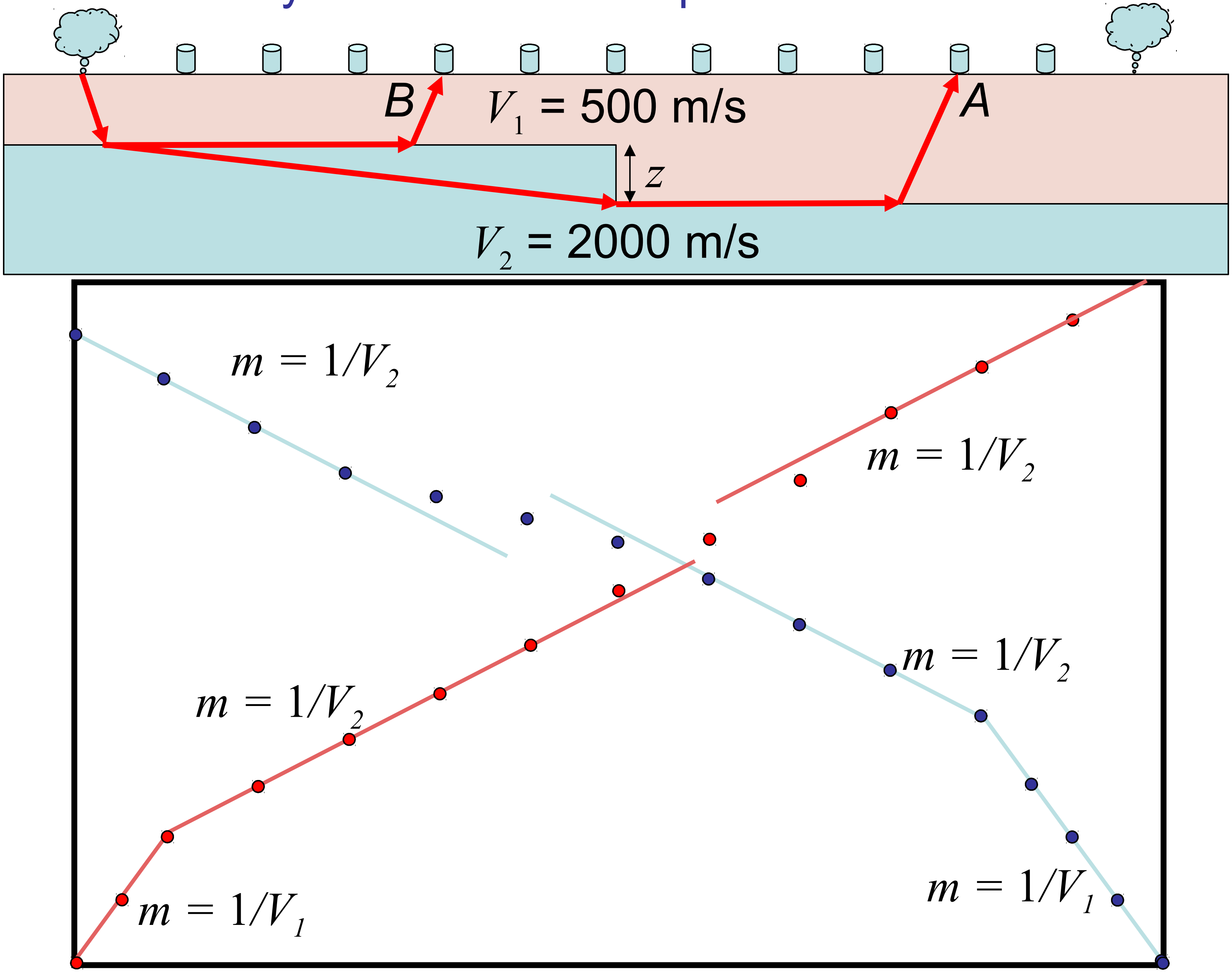


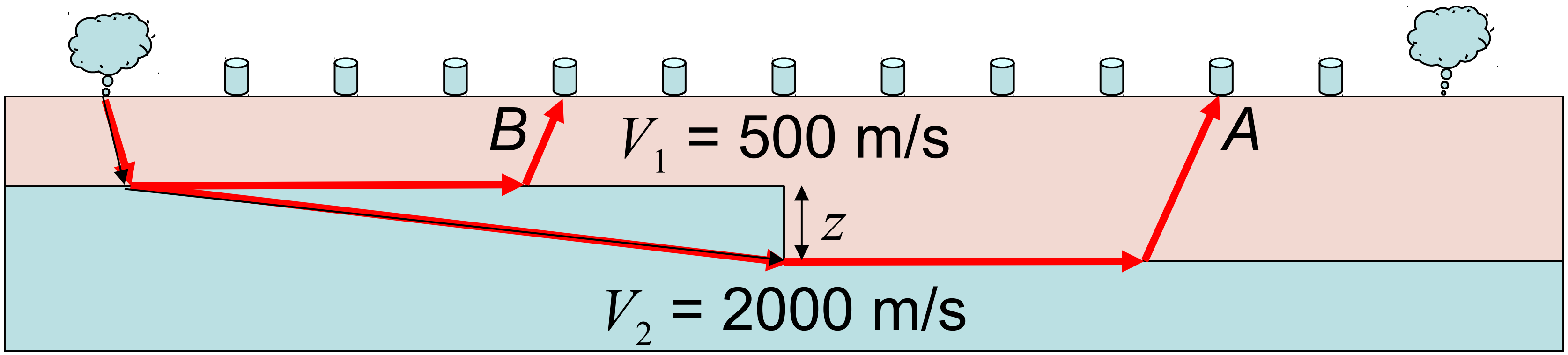




V_1 = 500 m/s
V_2 = 2000 m/s
V_3 = 1000 m/s
$m = 1/V_2$
$m = 1/V_3$
$m = 1/V_3$
$m = 1/V_2$
$m = 1/V_1$
$m = 1/V_1$

What if the layer interface is not planar?





Basically have a difference in the intercept times on one part of the geophone string relative to the other:

$$t_{0a} - t_{0b} = \frac{z\cos i_c}{V_1} = \frac{z\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

$$z = \frac{\left(t_{0a} - t_{0b}\right) V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$