Last time: The Refraction Method

- For a ***single horizontal layer over a halfspace***, observed travel-times for ***direct*** and ***refracted*** arrivals:

$$t = \frac{x}{V_1} \qquad t = \frac{x}{V_2} + \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

give the velocities and layer thickness from $t = m_i x + b_i$ as:

$$V_1 = \frac{1}{m_1} \qquad V_2 = \frac{1}{m_2} \qquad h = \frac{b_2 V_1 V_2}{2\sqrt{V_2^2 - V_1^2}}$$

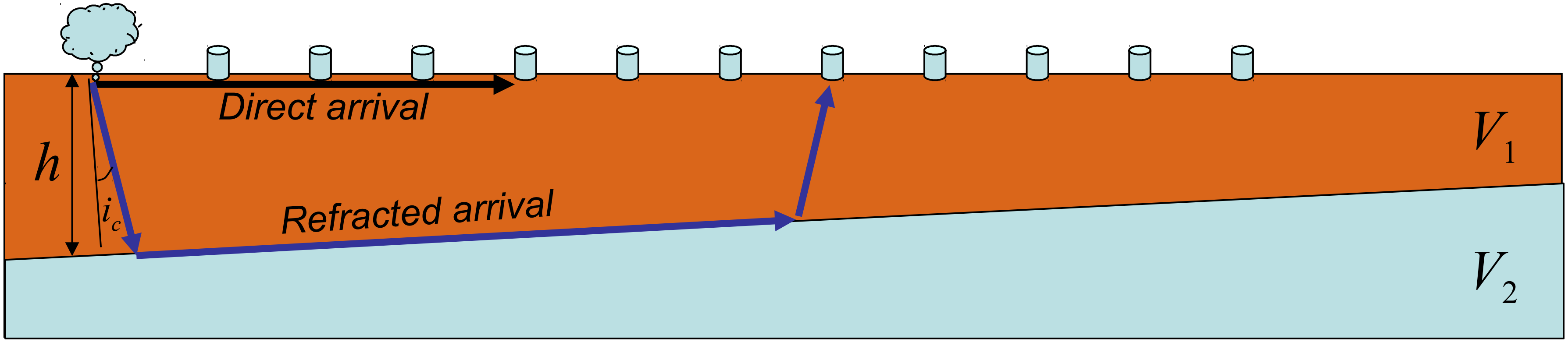
- The ***crossover distance***, x_{co} can be used in experiment design, to ensure adequate geophone sampling:

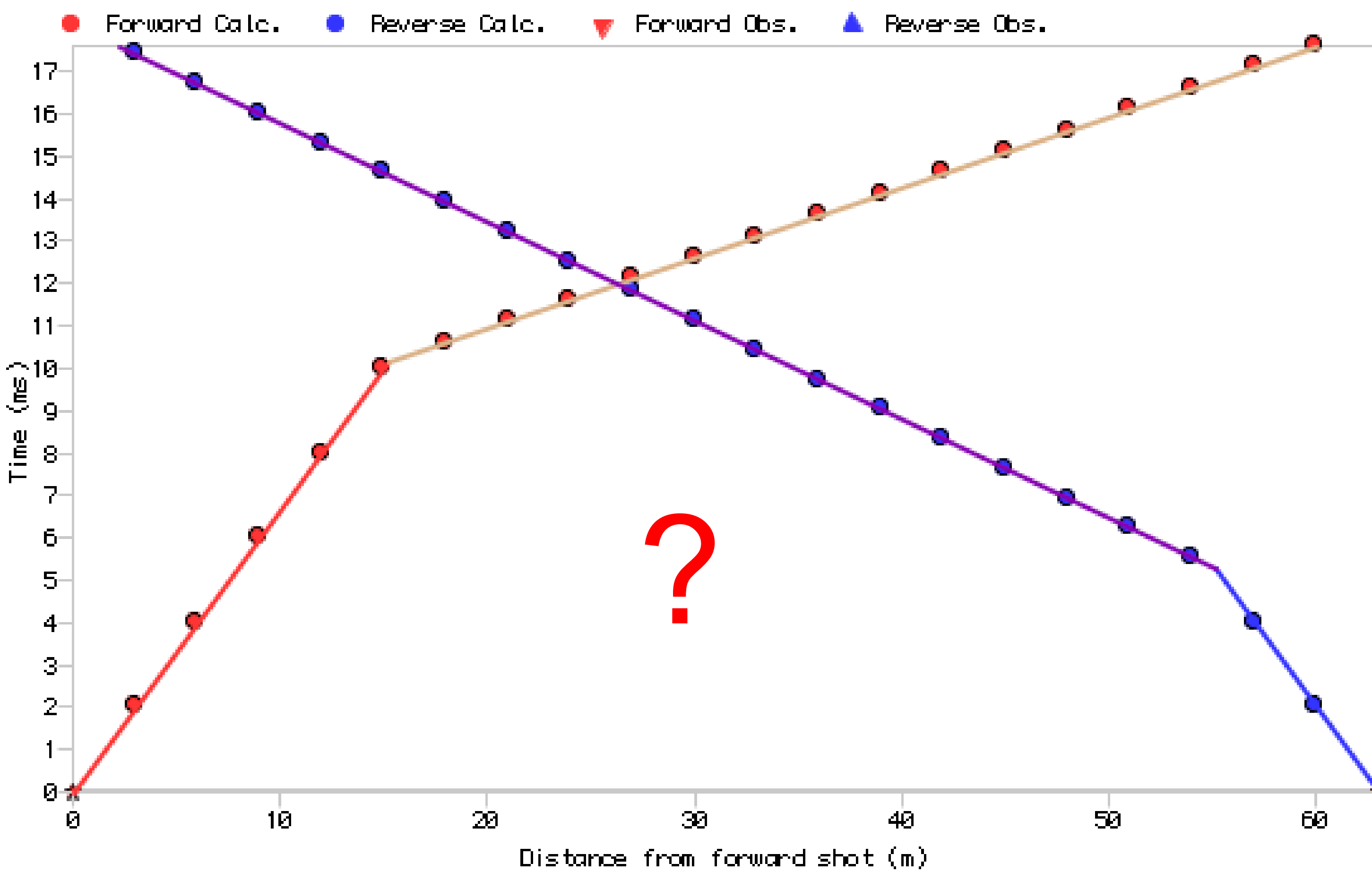
$$x_{co} = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}}$$

- For an n-layer Earth:

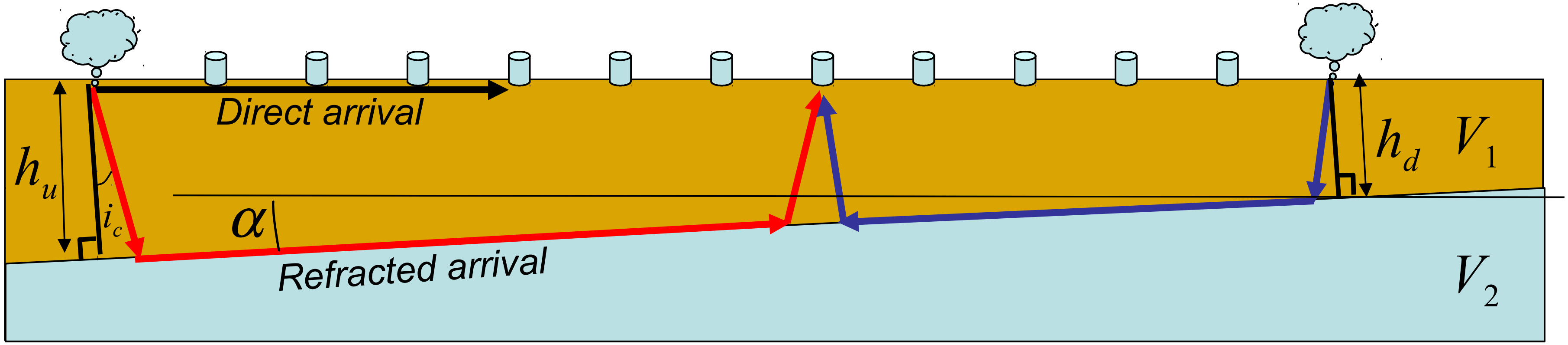
$$t = \frac{x}{V_n} + \sum_{i=1}^{n-1} \frac{2h_i\sqrt{V_n^2 - V_i^2}}{V_n V_i}$$

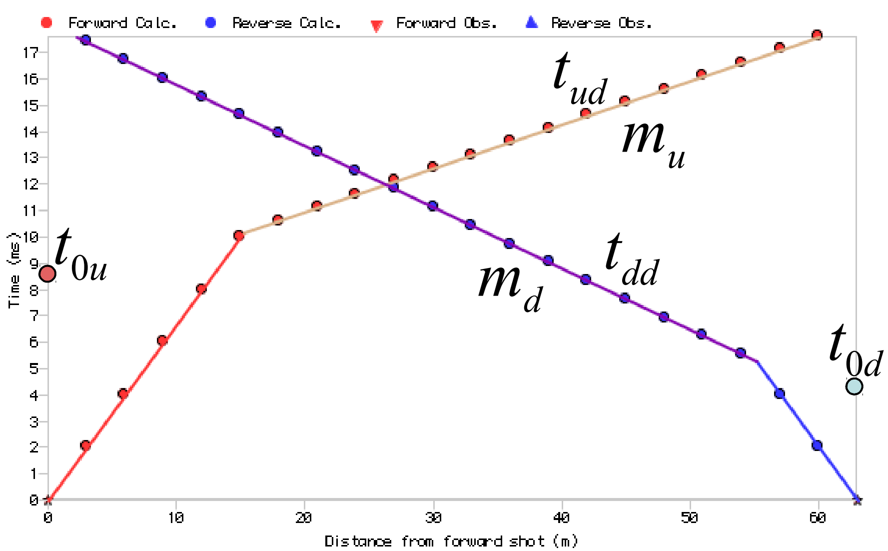
Read for Monday: ***Burger*** 106-141 (§3.7-3.11)





Model travel times





Model travel times

Refraction from a ***dipping interface***:

Now thickness changes with offset, so slope is no longer = inverse of layer velocity. Refer to as "***apparent velocity***", and this is why we reverse our shots!

After some trig & algebra, we get:

$$t_{ud} = \frac{2h_u \cos i_c}{V_1} + \frac{x}{V_1}\sin(i_c - \alpha) \qquad t_{dd} = \frac{2h_d \cos i_c}{V_1} + \frac{x}{V_1}\sin(i_c + \alpha)$$

Like before, we have the ***observed*** travel-times and want to know the ***model parameters***. We know V_1 from the direct arrival, & we know the slopes are:

$$m_u = \frac{\sin(i_c - \alpha)}{V_1} \qquad m_d = \frac{\sin(i_c + \alpha)}{V_1}$$

then we can solve for velocity V_2:

$$i_c = \frac{\sin^{-1}(V_1 m_u) + \sin^{-1}(V_1 m_d)}{2} \left\{ = \sin^{-1}\left(\frac{V_1}{V_2}\right) \right\} \Rightarrow \quad V_2 = V_1 \Big/ \sin\left\{ \frac{\sin^{-1}(V_1 m_u) + \sin^{-1}(V_1 m_d)}{2} \right\}$$

and angle: $$\alpha = \frac{\sin^{-1}(V_1 m_d) - \sin^{-1}(V_1 m_u)}{2}$$

The normal thicknesses use intercept times t_{0u}, t_{0d}:

$$h_d = \frac{t_{0d} V_1}{2\cos i_c} \qquad h_u = \frac{t_{0u} V_1}{2\cos i_c}$$

(Can get vertical distance from shot to top of layer by dividing h_d or h_u by $\cos\alpha$).

v_c

Dipping Layer t-x equations: Graphical solution

$$\sin(\theta_c) = \frac{V_1}{V_2}$$

We have:

$$t_D = \frac{x}{V_1} \qquad \Rightarrow \qquad m_0 = 1/V_1$$

$$t_u = \frac{x}{V_1}\sin(\theta_c - \alpha) + \frac{2h_u\cos(\theta_c)}{V_1} = m_u x + t_{0u}$$

$$t_d = \frac{x}{V_1}\sin(\theta_c + \alpha) + \frac{2h_d\cos(\theta_c)}{V_1} = m_d x + t_{0d}$$

(1) $V_1 = \frac{1}{m_0} = \frac{x}{t}(x \le x_{co})$

(2) $\alpha = \frac{\sin^{-1}\left(\frac{m_d}{m_0}\right) - \sin^{-1}\left(\frac{m_u}{m_0}\right)}{2}$

(3) $\theta_c = \frac{\sin^{-1}\left(\frac{m_d}{m_0}\right) + \sin^{-1}\left(\frac{m_u}{m_0}\right)}{2}$

and

(4)

$$V_2 = \frac{V_1}{\sin(\theta_c)} \qquad (a)$$

$$h_d = \frac{t_{0d}}{2m_0\cos(\theta_c)} \qquad (b)$$

$$h_u = \frac{t_{0u}}{2m_0\cos(\theta_c)} \qquad (c)$$

Aside: Solving for parameters in this way is a simple form of ***inversion*** of the data

Most geophysical (& geological, & other) problems can be expressed as:

observations = some function of parameters

$$\vec{d} = F\left(\vec{m}\right)$$

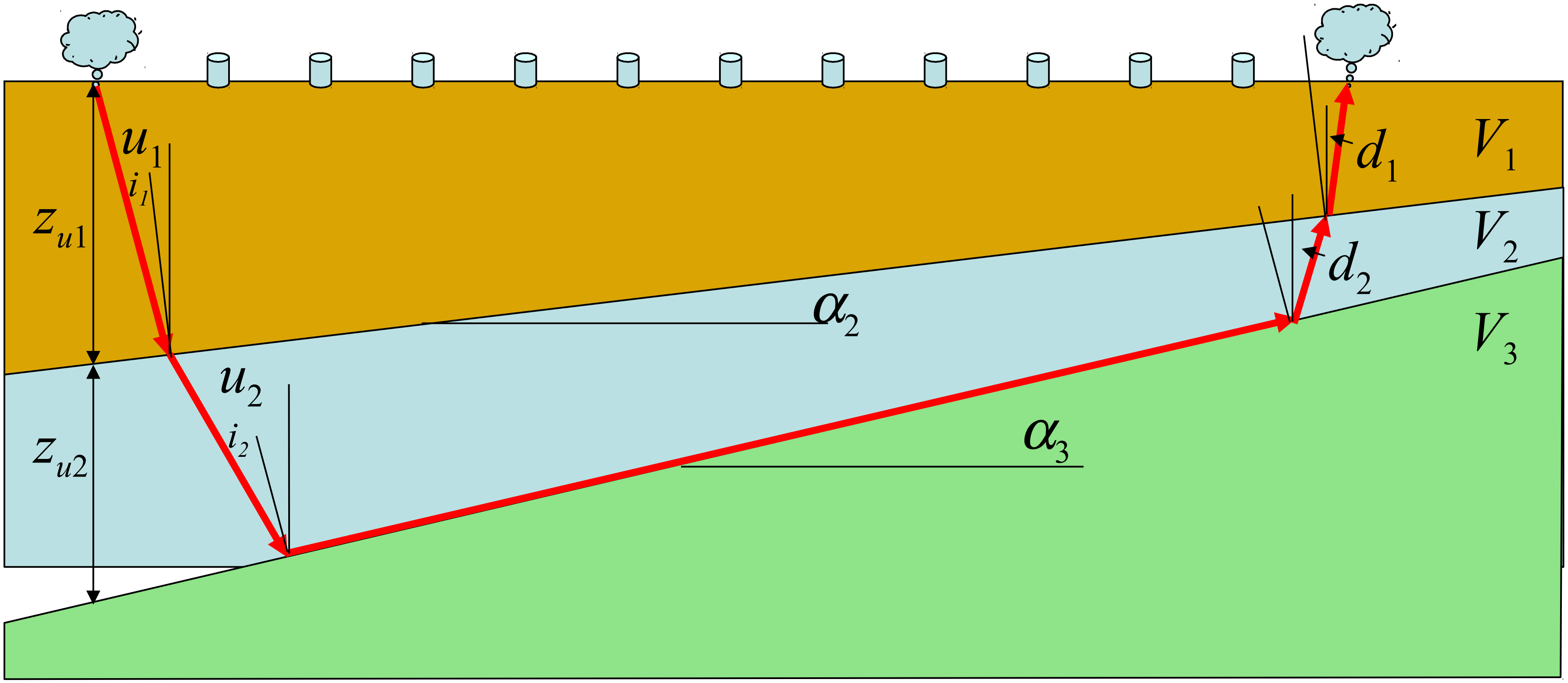
(arrow denotes plural: a vector)

Inverse theory, in which we seek to find

$$\vec{m} = F^{-1}\left(\vec{d}\right)$$

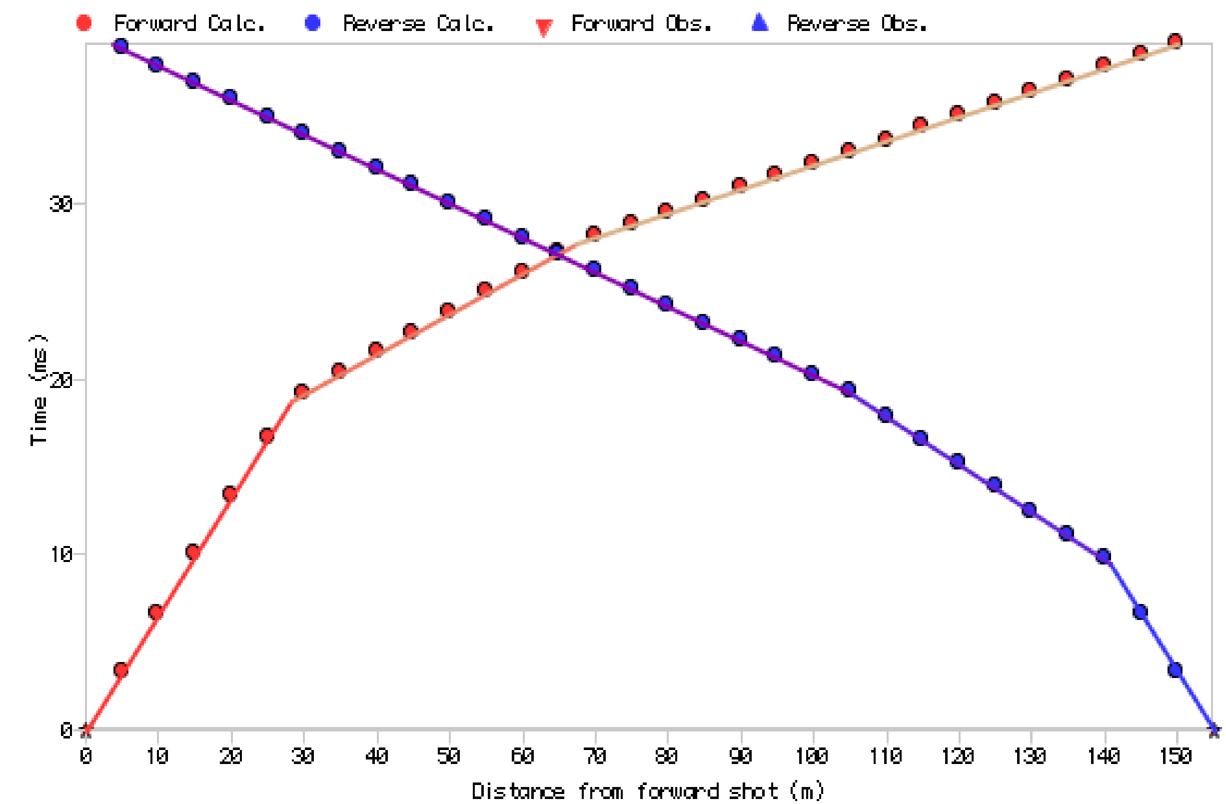
is a ***very important*** part of geophysics!

Equations for multiple dipping layers: Derived using an approach similar (but slightly more algebraically complicated) to that used for the multi-horizontal-layer case [*Adachi*]:



$$t_{ud} = \frac{x \sin d_1}{V_1} + \sum_{i=1}^{n-1} \frac{z_{ui}}{V_i}\left(\cos u_i + \cos d_i\right) \qquad t_{dd} = \frac{x \sin u_1}{V_1} + \sum_{i=1}^{n-1} \frac{z_{di}}{V_i}\left(\cos u_i + \cos d_i\right)$$

(These are the equations used in ***Refract***)



Model travel times