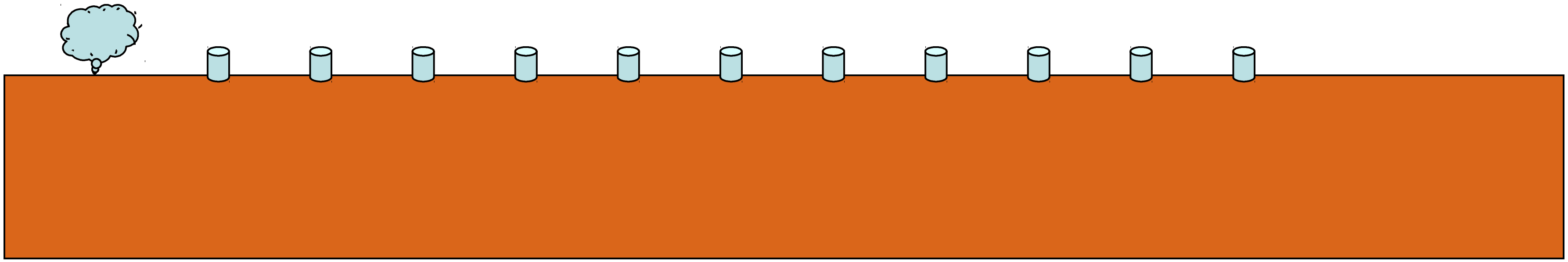
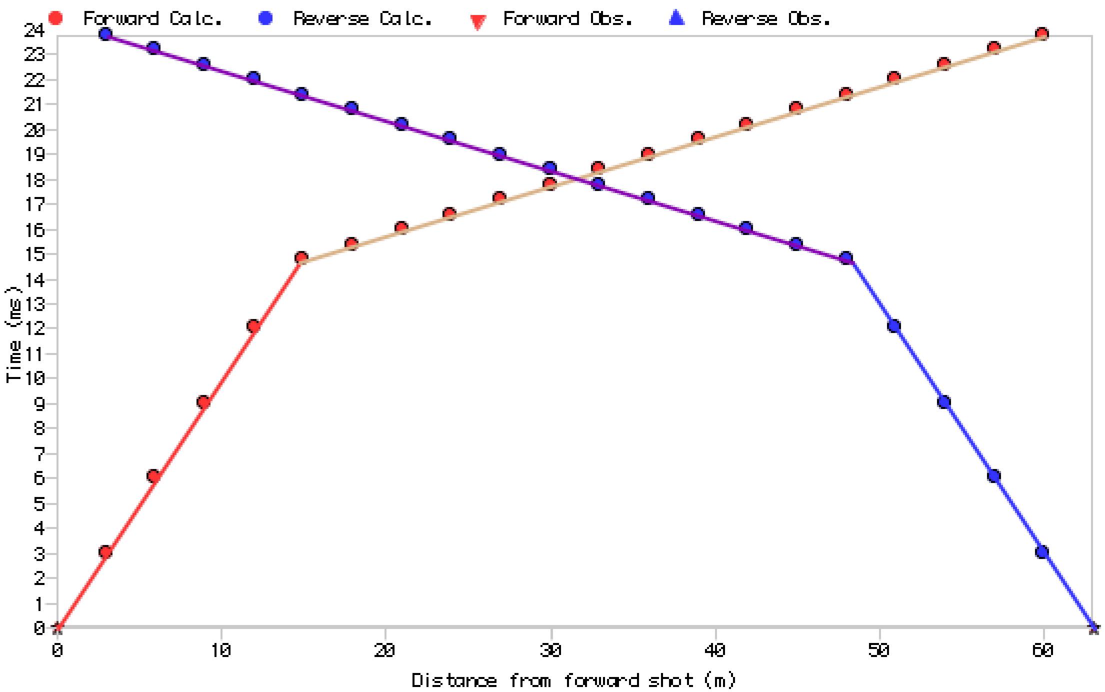
The Refraction Method:

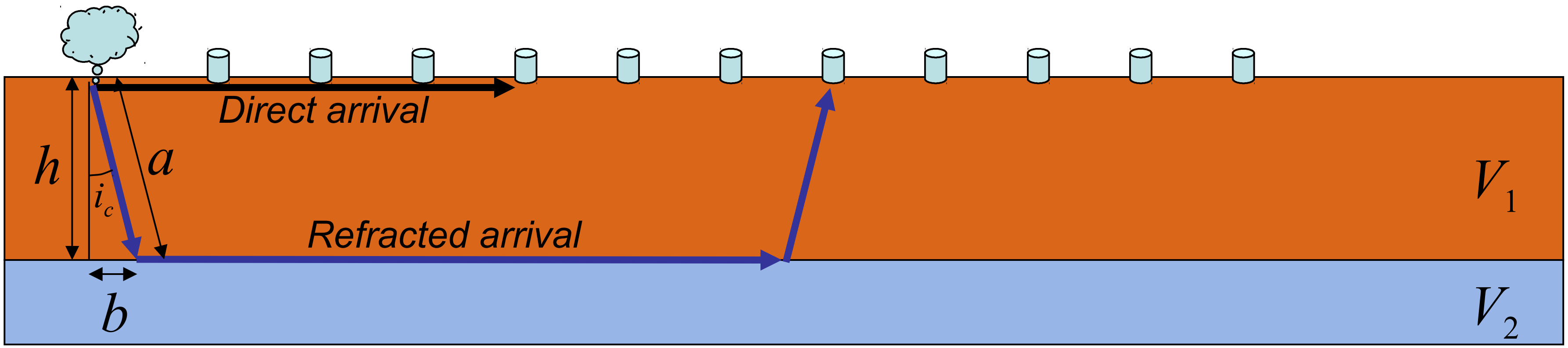
Recall first arriving seismic wave is the ***direct wave*** near the source, and ***refracted (head) waves*** further out:





Problem:

We start knowing little or nothing about the subsurface, and our objective is to use the ***observations*** to learn what's under our feet.



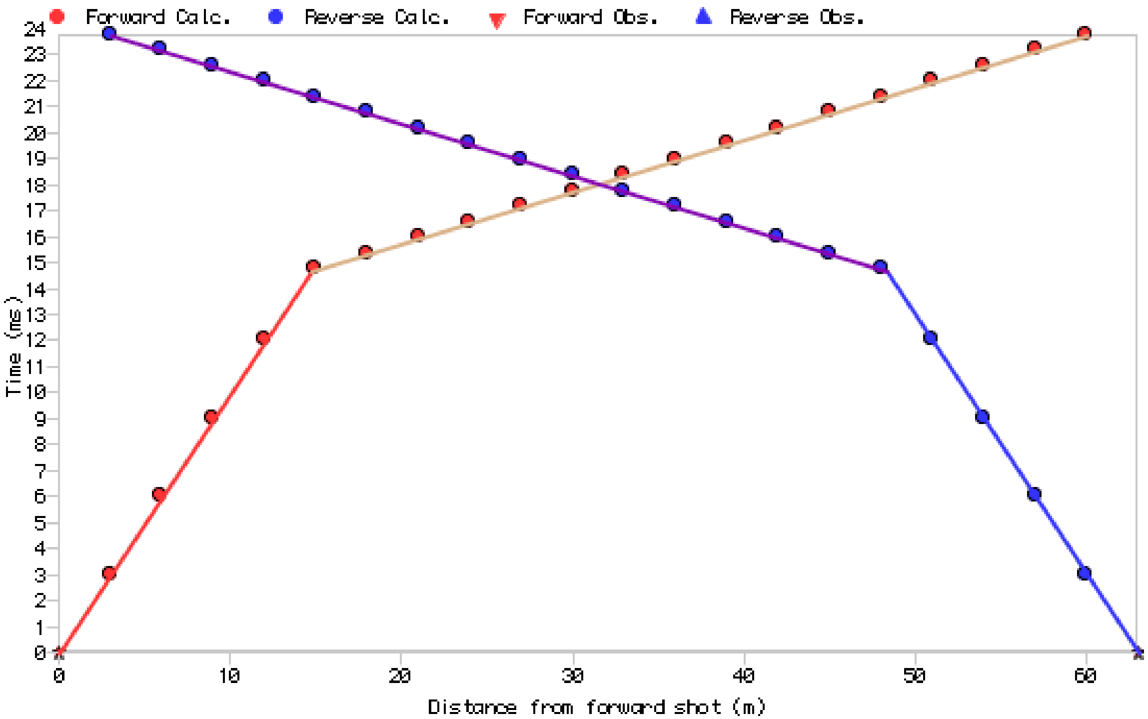
Consider the simple case of one horizontal layer over a half-space with higher velocity:

Direct wave travels horizontally at velocity V_1. On a plot of time vs. distance, arrivals are a straight line with ***slope*** $\Delta t/\Delta x = 1/V_1$ and ***intercept*** $t = 0$, i.e., $t = x/V_1$.

Refracted wave travels horizontally at velocity V_2 at the top of layer 2, so a line with ***slope*** $1/V_2$. The ***intercept*** now adds the time traveled in the first layer (& subtracts the time not traveled in the second layer), i.e., $t_0 = 2a/V_1 - 2b/V_2$. This wave has incident angle $i_c = \sin^{-1}(V_1/V_2)$, so

$$t = \frac{x}{V_2} + \frac{2a}{V_1} - \frac{2b}{V_2} = \frac{x}{V_2} + \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

(HW # 1-4)



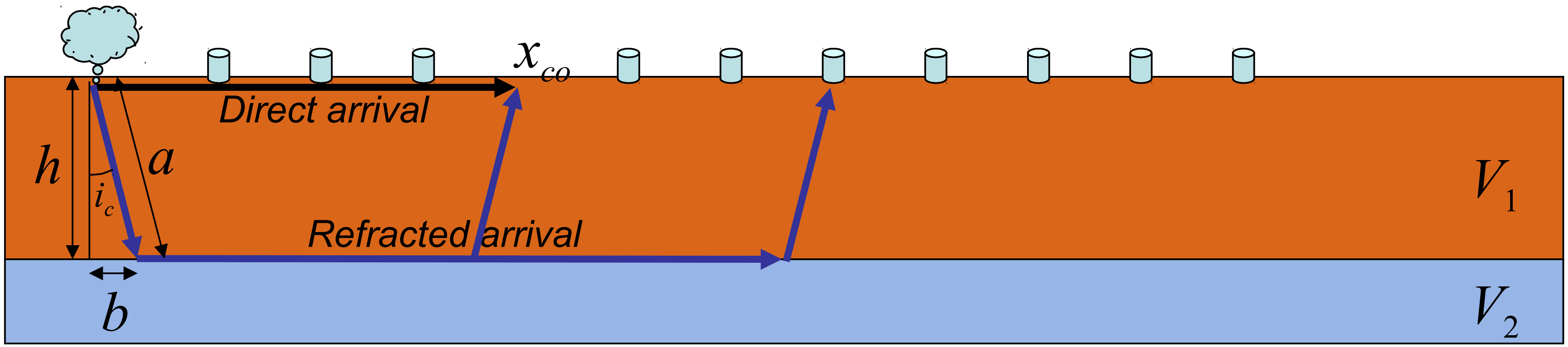
The mathematical simplicity of finding structure from slope & intercept of a line made refraction the tool of choice in early imaging studies!

So, can fit lines $t = m_i x + b_i$ to the first arrival travel-times…

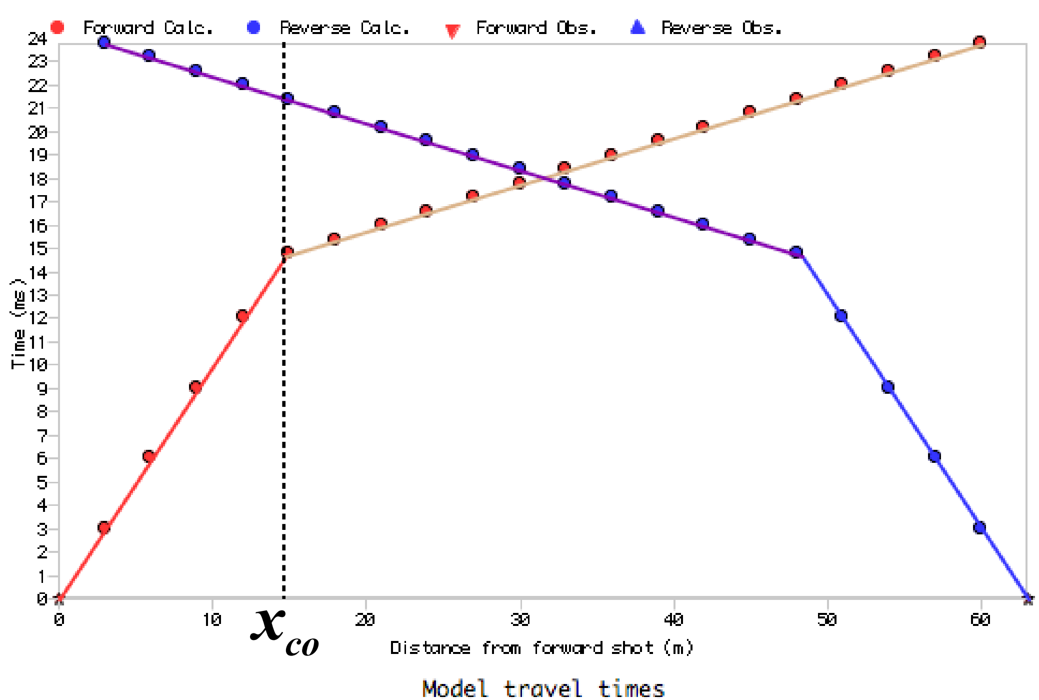
Then $V_1 = 1/m_1$, $V_2 = 1/m_2$, and

$$h = \frac{b_2 V_1 V_2}{2\sqrt{V_2^2 - V_1^2}}$$

In the plot above, $m_1 = 0.001$ s/m, $m_2 = 0.0002$ s/m, $b_2 = 0.0118$ s. What are the velocities & depth? **(Exercise)**



Practical note: The following works for a horizontal interface, and ***then only*** if there are enough geophones before & after the ***crossover distance*** x_{co} (where arrival times for the direct & refracted arrival are the same) :



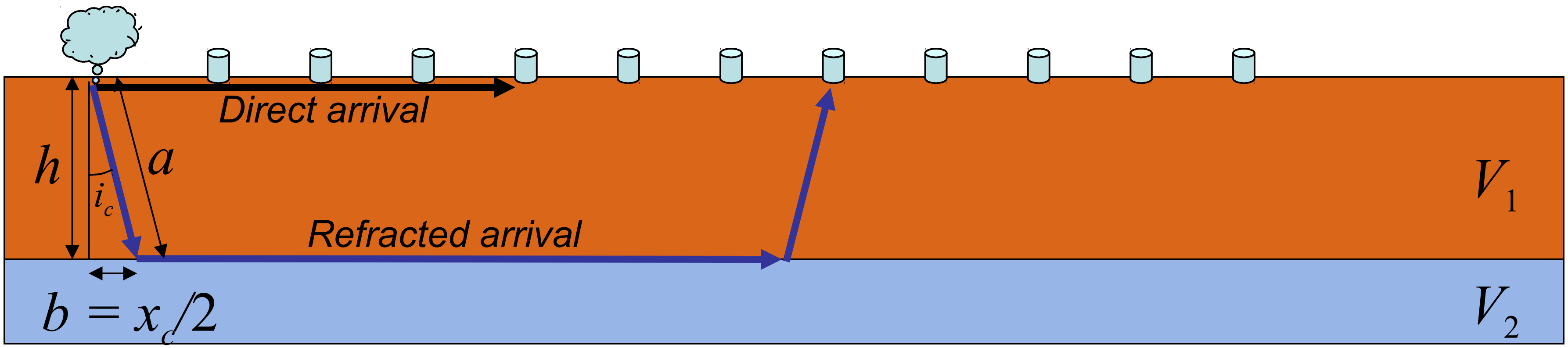
$$\frac{x_{co}}{V_1} = \frac{x_{co}}{V_2} + \frac{2h\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

So,

$$x_{co} = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}}$$

$$x_{co} = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}}$$

Spatial sampling is key to getting accurate estimates of thickness: Must have multiple geophones that sample each layer. Since we don’t know the structure when designing the experiment, this means lots of closely-spaced geophones… Where “close” is determined by minimum expected thickness and maximum expected velocity difference!

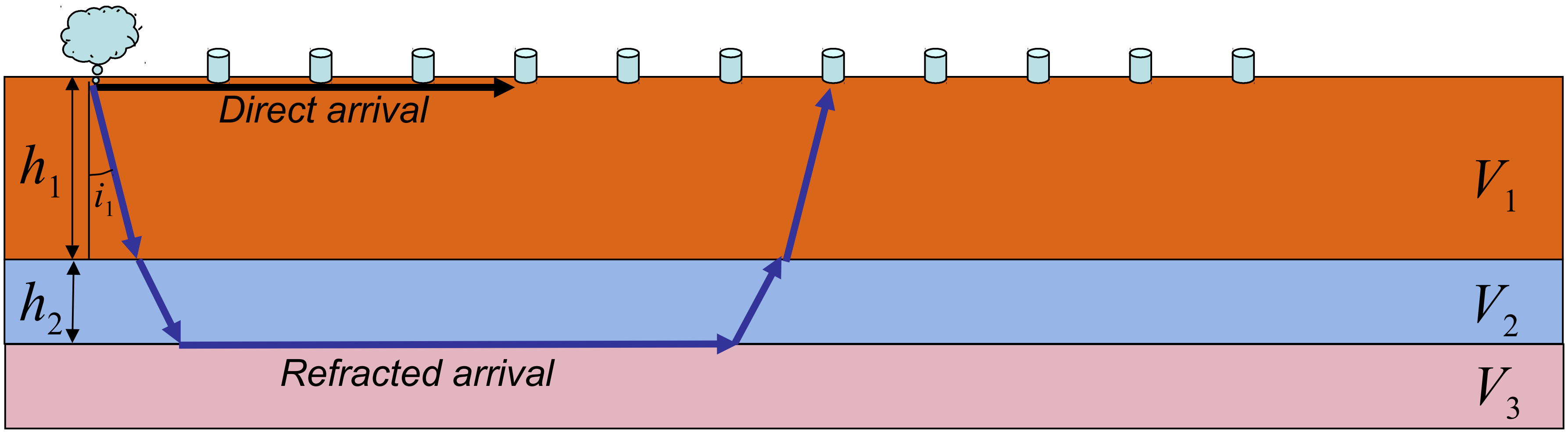


Another important term in refraction studies is the ***critical distance***, the horizontal distance traveled in layer 1 by a ray at the ***critical angle***, i_c :

$$x_c = 2b$$

$$\tan i_c = \frac{x_c/2}{h} \quad \Rightarrow \quad x_c = 2h \tan\left(\sin^{-1}\left(\frac{V_1}{V_2}\right)\right) = \frac{2hV_1}{\sqrt{V_2^2 - V_1^2}}$$

In situations where we have some approximate idea of the range of thicknesses & velocities we might expect to find, calculating the crossover and critical distances inform ***experiment design*** (i.e., how many geophones to use, offset & spacing).



For ***multiple layers*** we take advantage of Snell's Law:

$$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = \frac{\sin i_3}{V_3} \Rightarrow \sin i_c = \frac{V_2}{V_3}; \quad \sin i_1 = \frac{V_1}{V_3}$$

Via similar algebra & geometry to the one-layer case,

$$t = \frac{x}{V_3} + \frac{2h_1\sqrt{V_3^2 - V_1^2}}{V_3 V_1} + \frac{2h_2\sqrt{V_3^2 - V_2^2}}{V_3 V_2}$$

or, generalizing to n-layers:

$$t = \frac{x}{V_n} + \sum_{i=1}^{n-1} \frac{2h_i\sqrt{V_n^2 - V_i^2}}{V_n V_i}$$