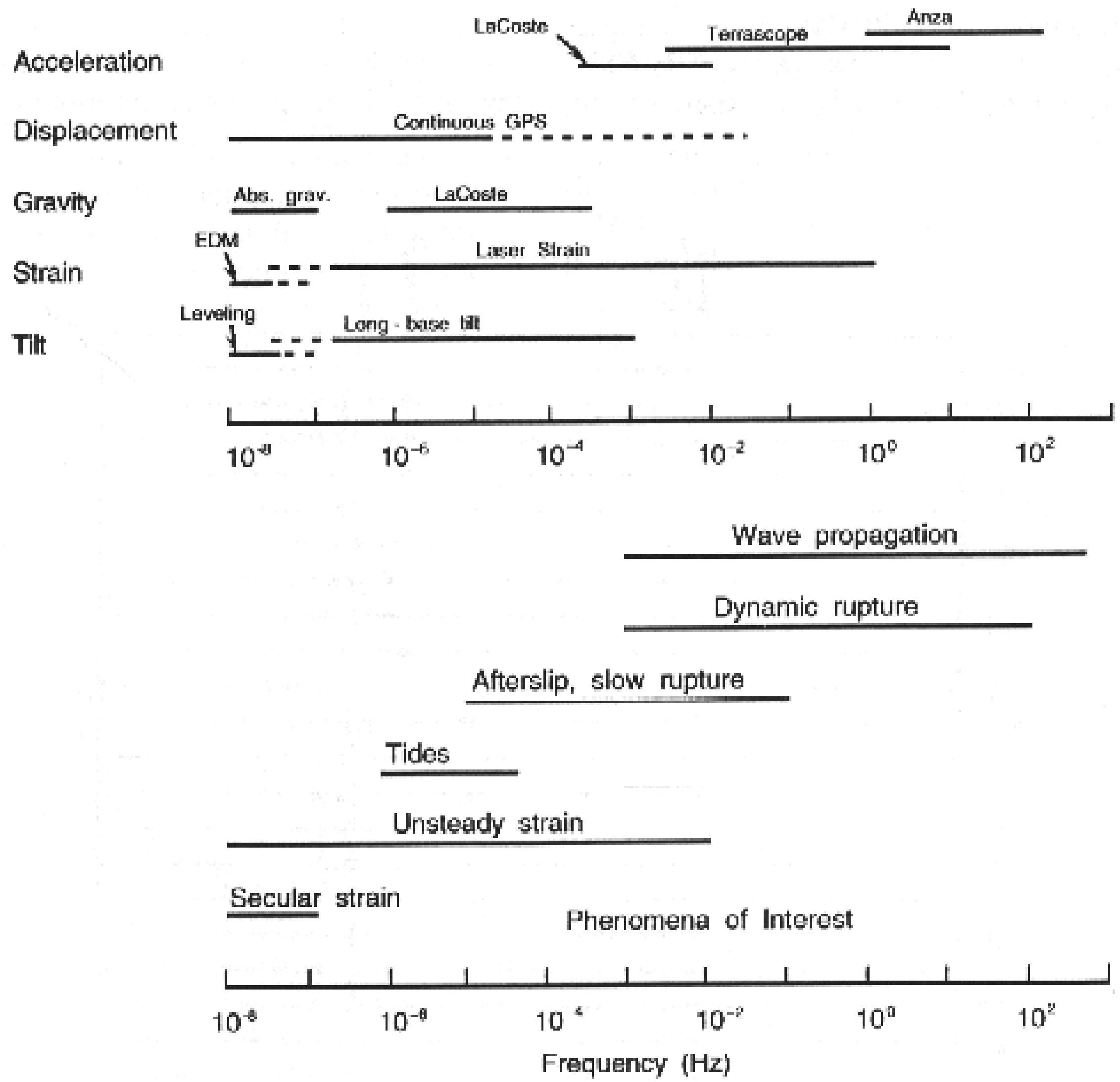
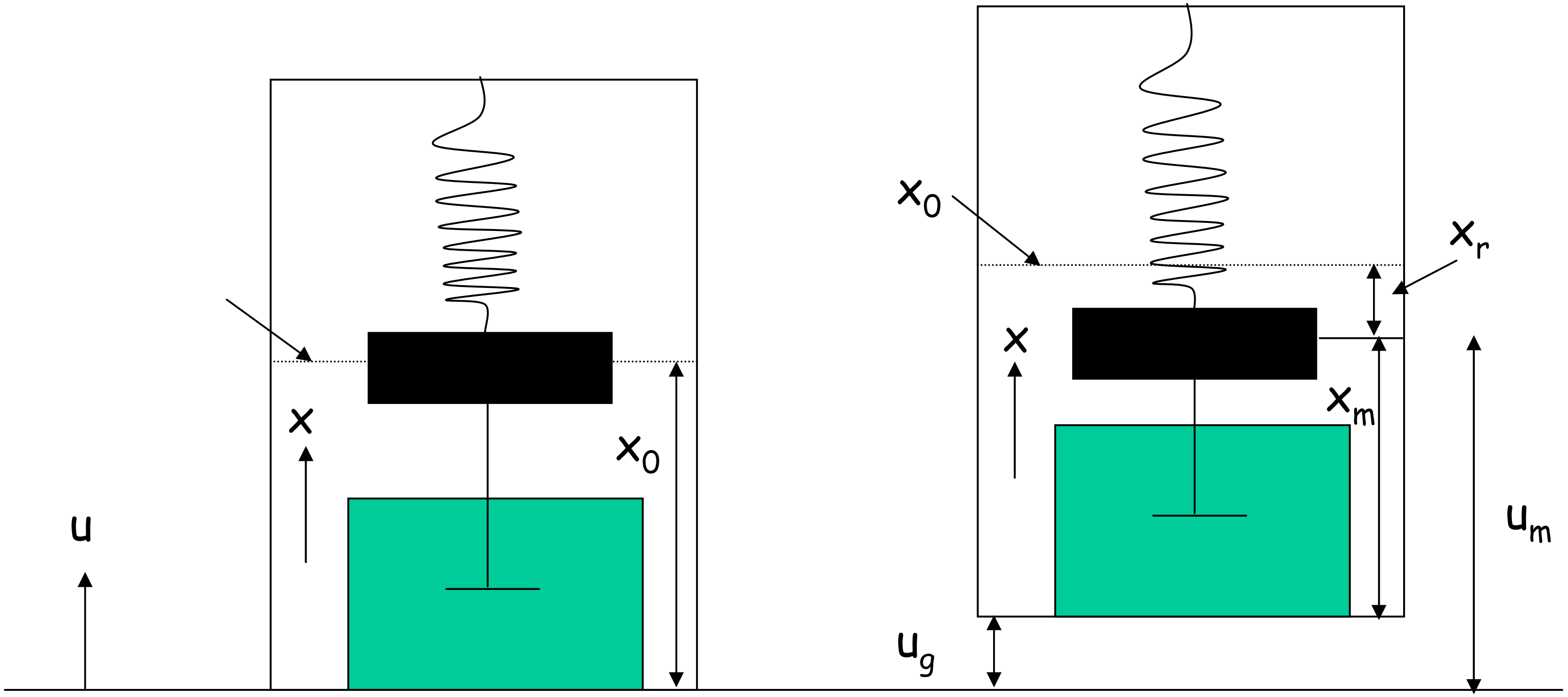
Time Scales in Seismology



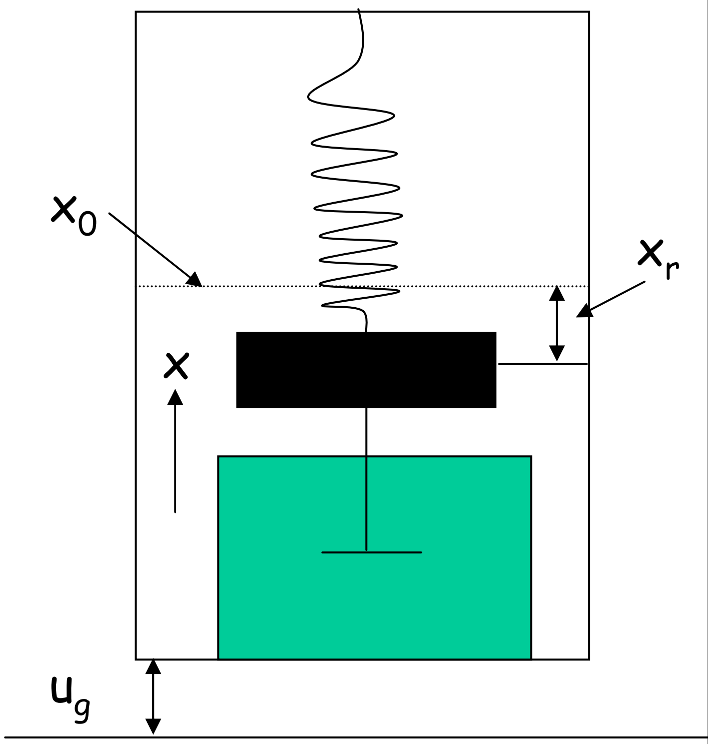
Seismometer – The basic Principles



u	ground displacement
x_r	displacement of seismometer mass
x_0	mass equilibrium position

Seismometer – The basic Principles

The motion of the seismometer mass as a function of the ground displacement is given through a differential equation resulting from the equilibrium of forces (in rest):



$F_{spring} + F_{friction} + F_{gravity} = 0$

for example

F_{sprin}=-k x, k spring constant

$F_{friction}$=-D $\dot{x}$, D friction coefficient

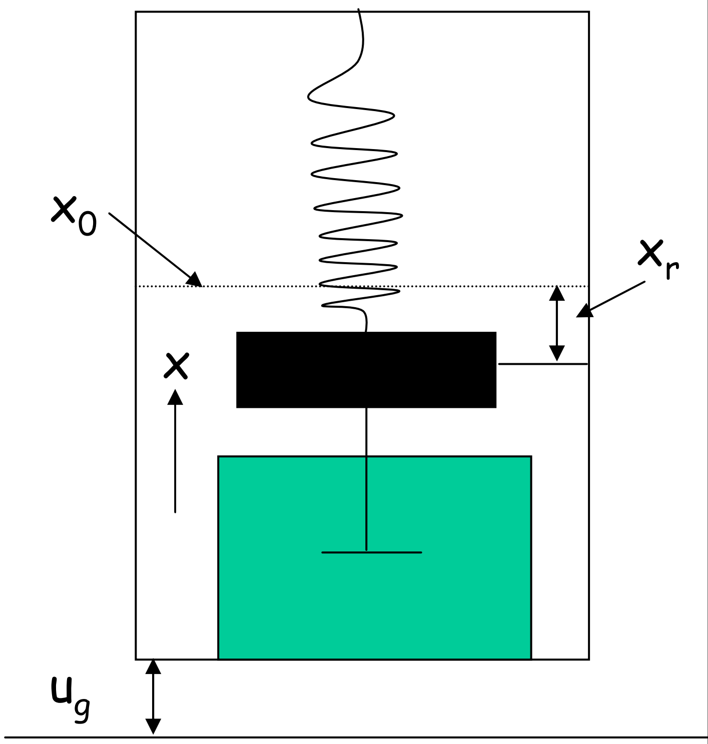
$F_{gravity}$=-m$\ddot{u}$, m seismometer mass

Seismometer – The basic Principles

using the notation introduced the equation of motion for the mass is

$$\ddot{x}_r(t) + 2\varepsilon\dot{x}_r(t) + \varpi_0^2 x_r(t) = -\ddot{u}_g(t)$$

$$\varepsilon = \frac{D}{2m} = h\varpi_0, \qquad \varpi_0^2 = \frac{k}{m}$$

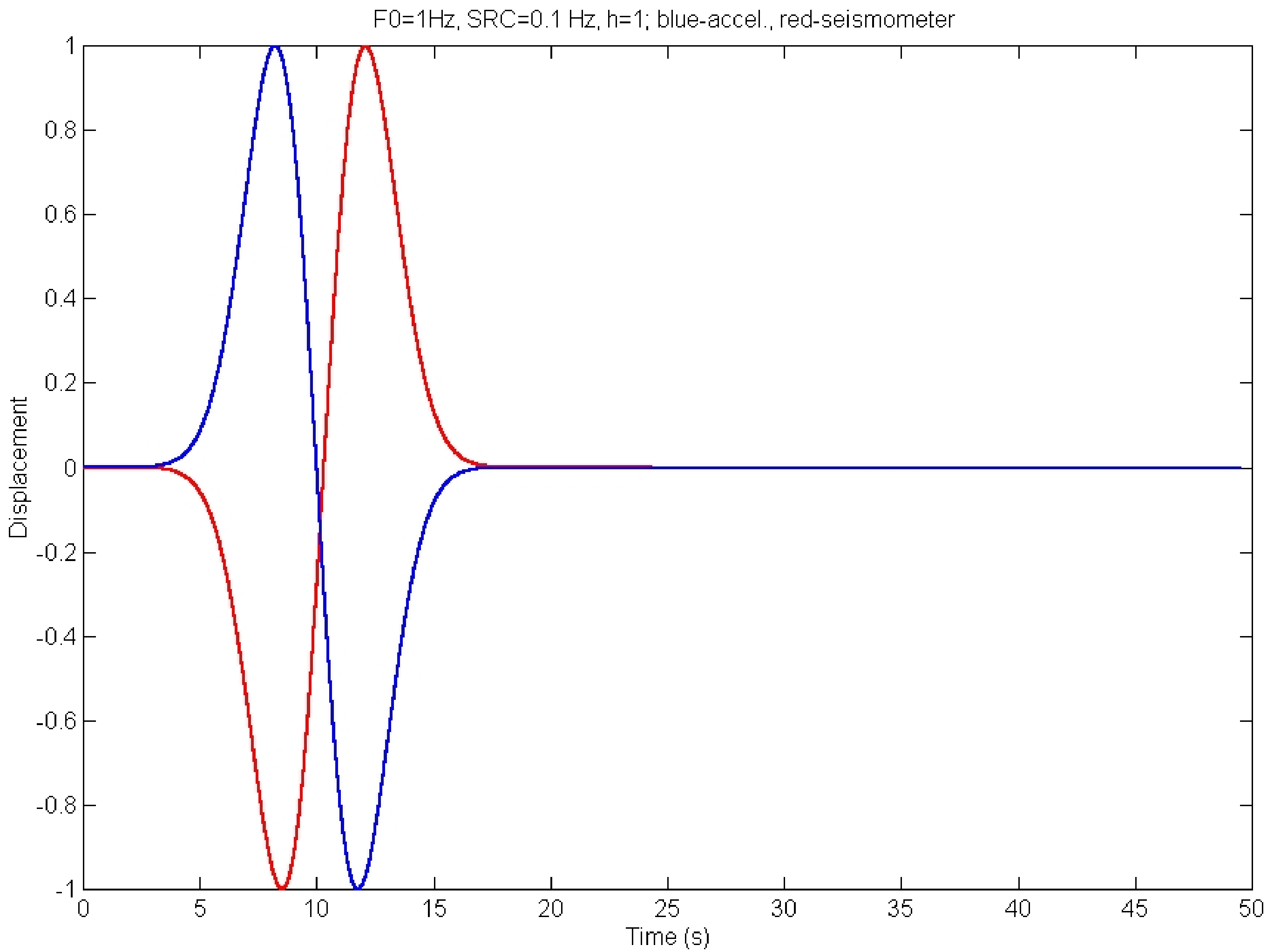


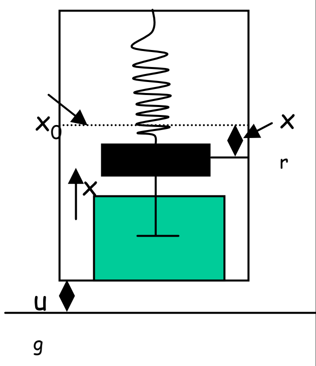
From this we learn that:

- for slow movements the acceleration and velocity becomes negligible, the seismometer records ground acceleration

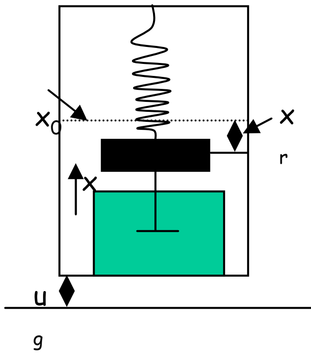
- for fast movements the acceleration of the mass dominates and the seismometer records ground displacement

Seismometer - examples





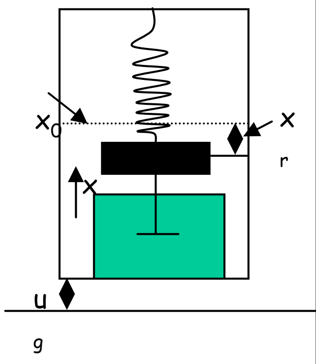
Seismometer – Questions



1. How can we determine the damping properties from the observed behavior of the seismometer?

2. How does the seismometer amplify the ground motion? Is this amplification frequency dependent?

We need to answer these question in order to determine what we really want to know:
The ground motion.



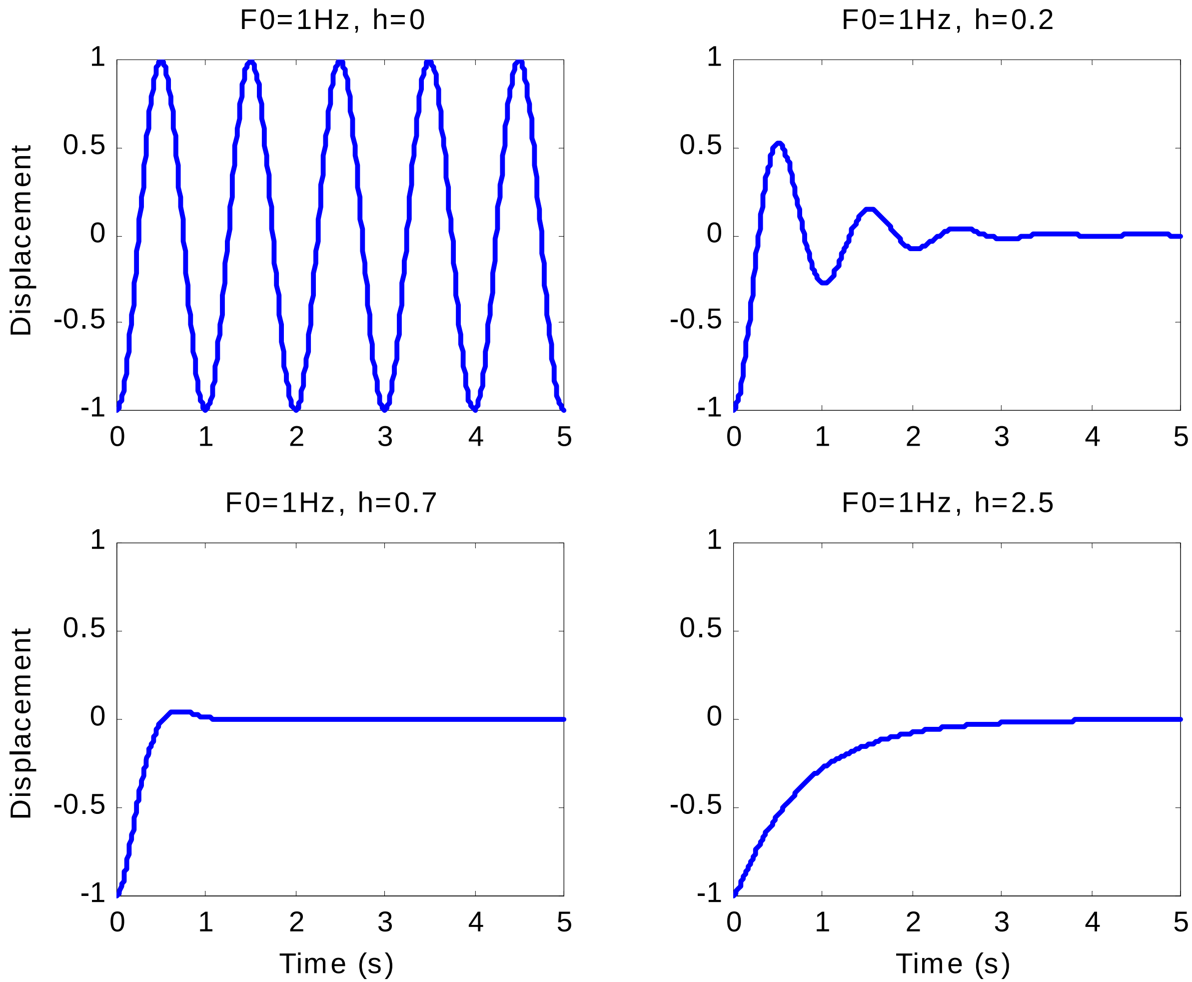
1. How can we determine the damping properties from the observed behavior of the seismometer?

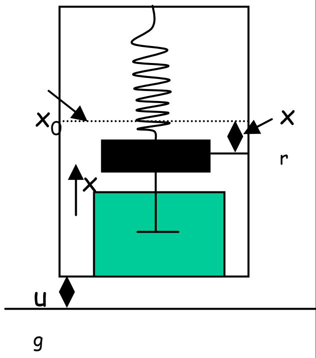
$$\ddot{x}_r(t) + h\varpi_0 \dot{x}_r(t) + \varpi_0^2 x_r(t) = 0$$

$$x_r(0) = x_0, \qquad \dot{x}_r(0) = 0$$

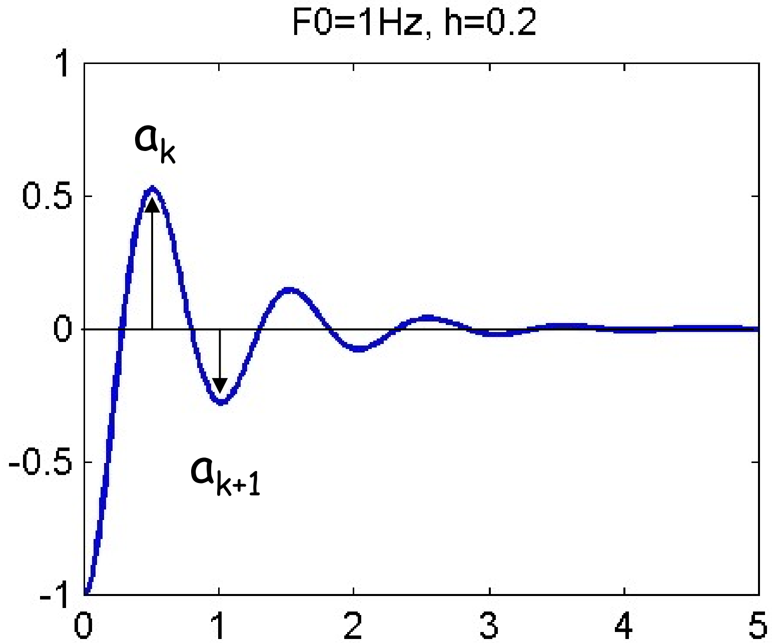
we release the seismometer mass from a given initial position and let is swing. The behavior depends on the relation between the frequency of the spring and the damping parameter. If the seismometers oscillates, we can determine the damping coefficient h.

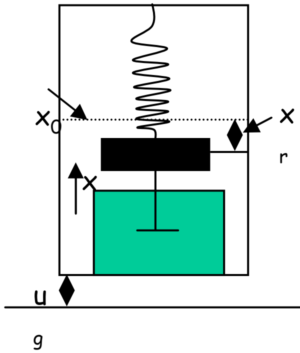
Seismometer - Release Test





Seismometer – Release Test





The damping coefficients can be determined from the amplitudes of consecutive extrema a_k and a_{k+1}

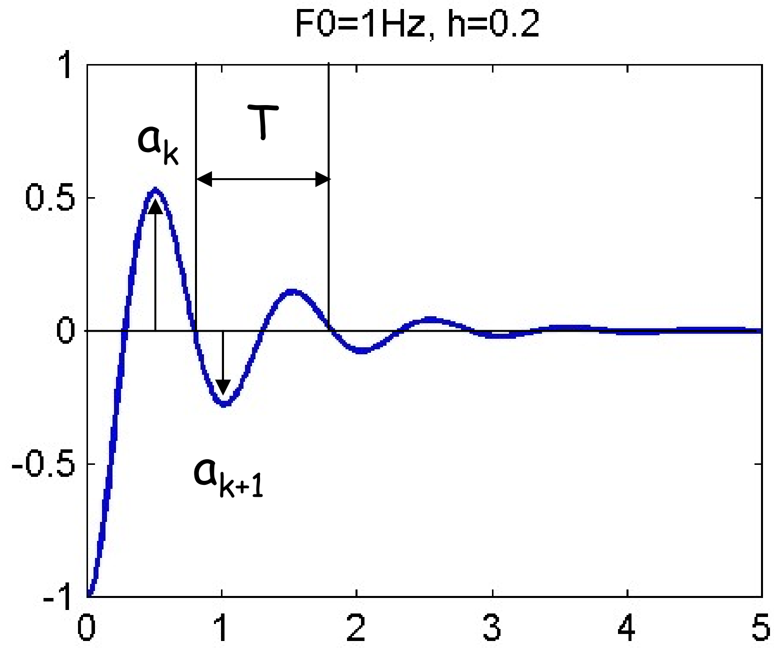
We need the logarithmic decrement Λ

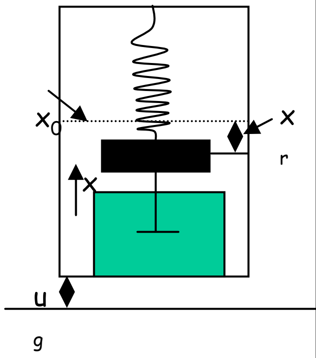
$$\Lambda = 2\ln\left(\frac{a_k}{a_{k+1}}\right)$$

The damping constant h can then be determined through:

$$h = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}}$$

Seismometer - Frequency

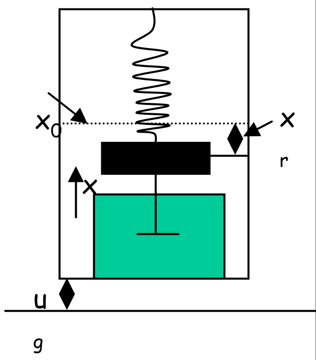




The period T with which the seismometer mass oscillates depends on h and (for h<1) is always larger than the period of the spring T_0:

$$T = \frac{T_0}{\sqrt{1-h^2}}$$

Seismometer – Response Function



2. How does the seismometer amplify the ground motion? Is this amplification frequency dependent?

To answer this question we excite our seismometer with a monofrequent signal and record the response of the seismometer:

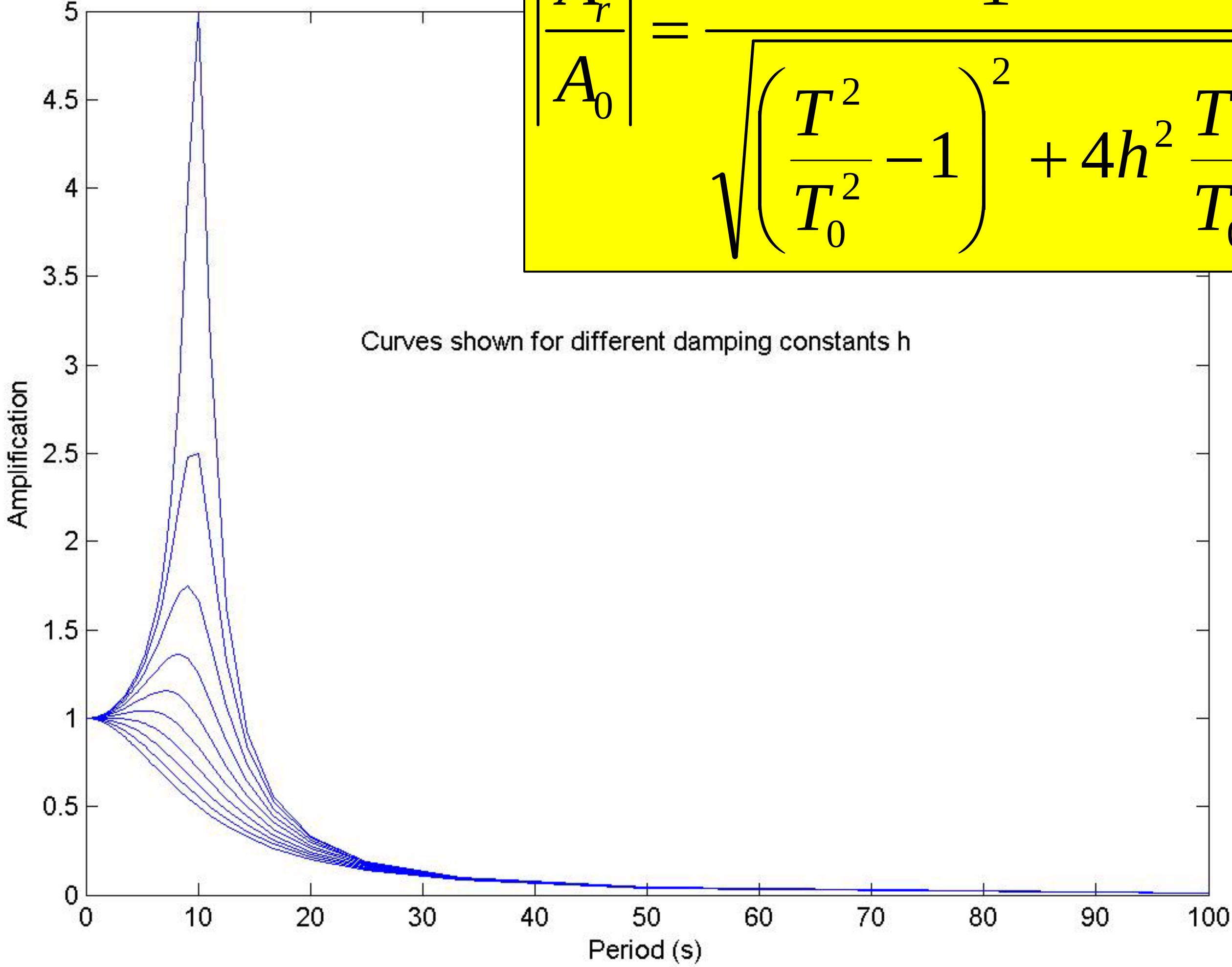
$$\ddot{x}_r(t) + h\varpi_0\dot{x}_r(t) + \varpi_0^2 x_r(t) = \varpi^2 A_0 e^{i\varpi t}$$

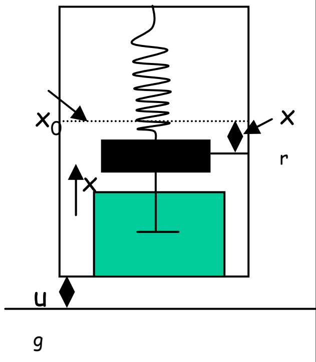
the amplitude response A_r of the seismometer depends on the frequency of the seismometer w_0, the frequency of the excitation w and the damping constant h:

$$\left|\frac{A_r}{A_0}\right| = \frac{1}{\sqrt{\left(\frac{T^2}{T_0^2} - 1\right)^2 + 4h^2\frac{T^2}{T_0^2}}}$$

Seismometer - Response Function

$$\left|\frac{A_r}{A_0}\right| = \frac{1}{\sqrt{\left(\frac{T^2}{T_0^2}-1\right)^2 + 4h^2\frac{T^2}{T_0^2}}}$$





(Relative) Dynamic range

What is the precision of the sampling of our physical signal in amplitude?

Dynamic range: the ratio between largest measurable amplitude A_{max} to the smallest measurable amplitude A_{min}.

The unit is Decibel (dB) and is defined as the ratio of two power values (and power is proportional to amplitude square)

In terms of amplitudes

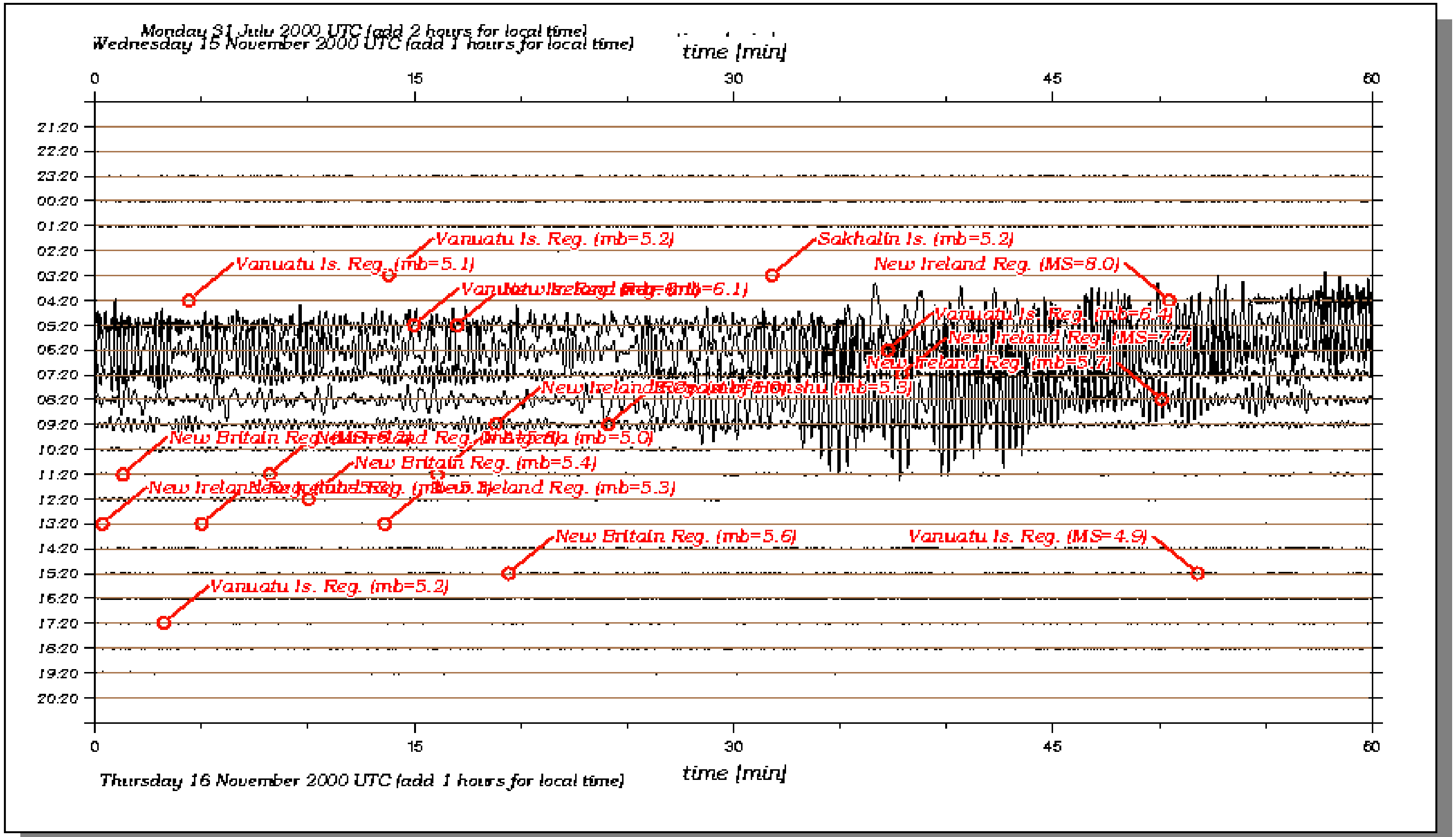
Dynamic range = $20 \log_{10}(A_{max}/A_{min})$ dB

Example: with 1024 units of amplitude (A_{min}=1, A_{max}=1024)

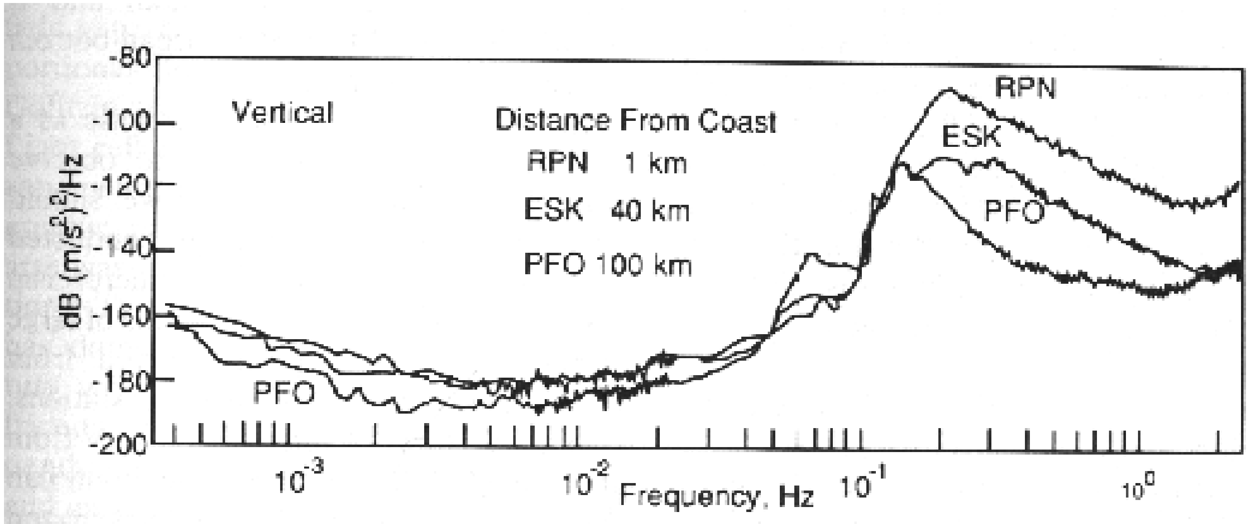
$$20 \log_{10}(1024/1) \text{ dB} \approx 60 \text{ dB}$$

Signal and Noise

Almost all signals contain noise. The signal-to-noise ratio is an important concept to consider in all geophysical experiments. Can you give examples of noise in the various methods?

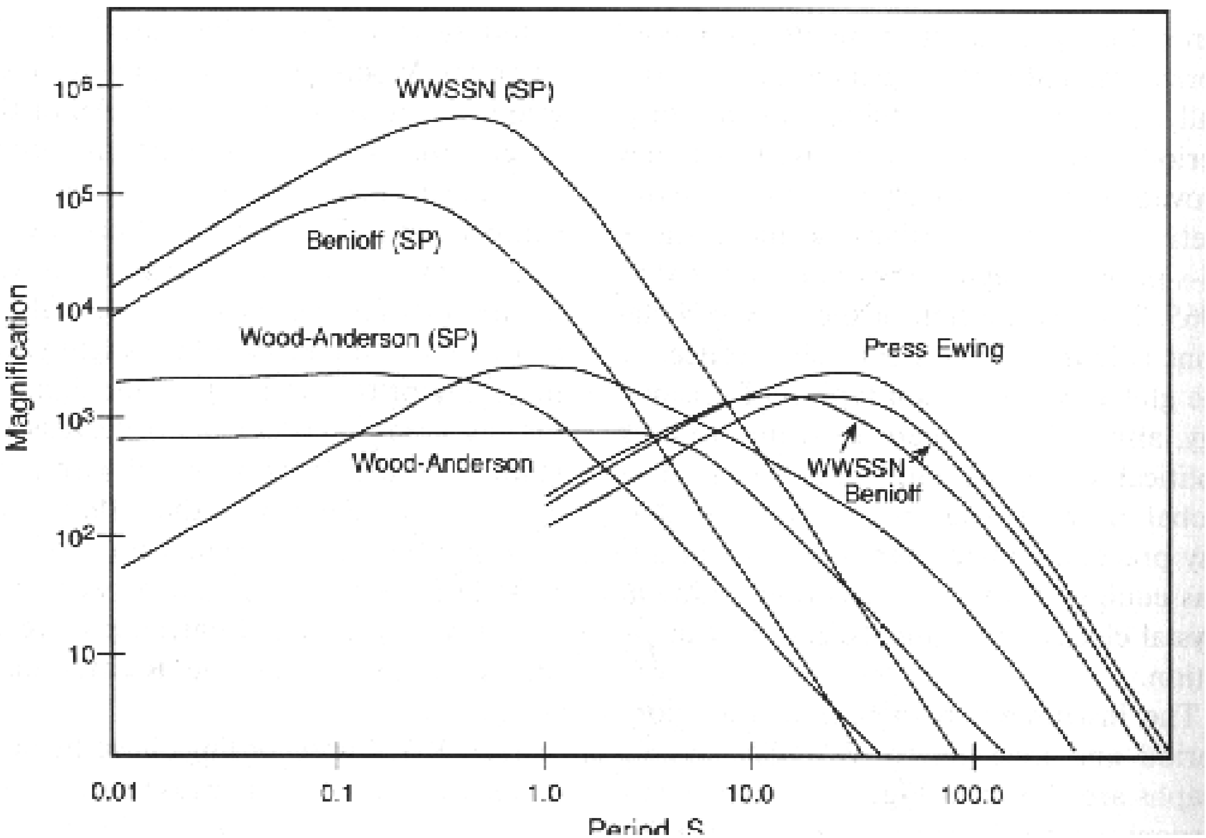


Seismic Noise

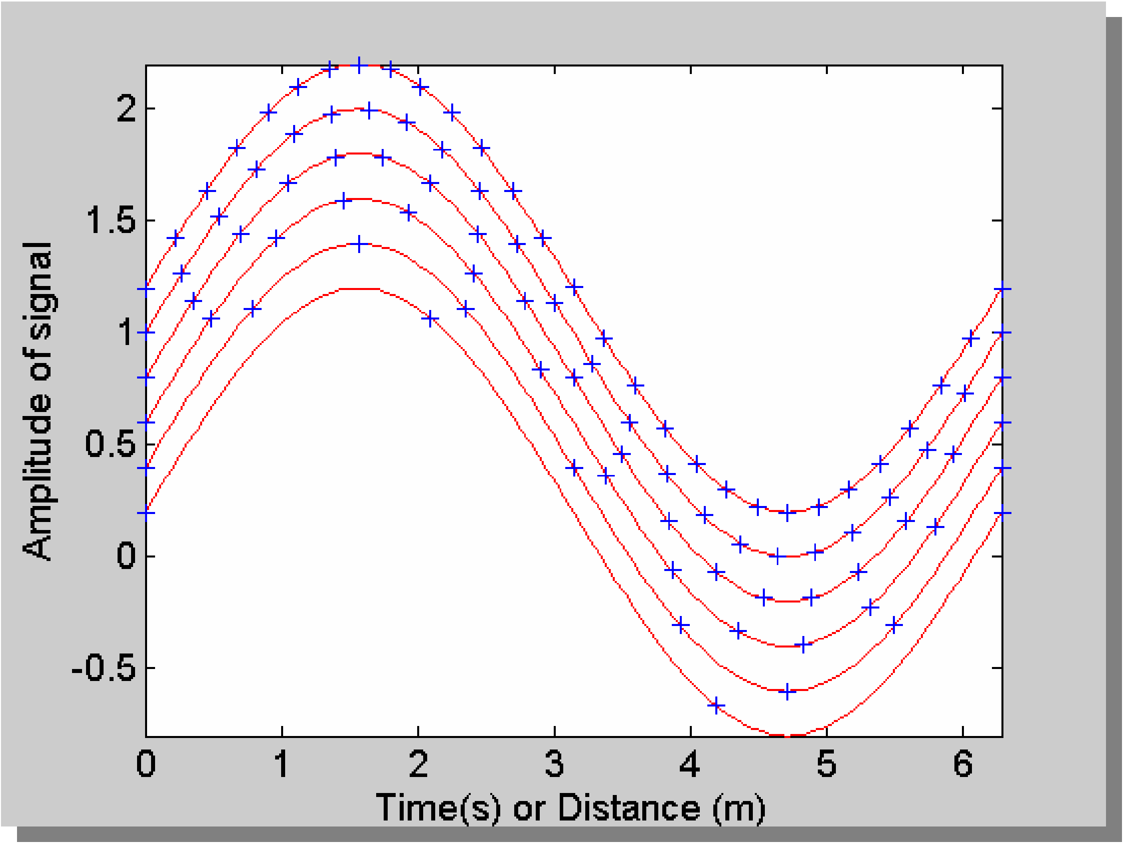


Observed seismic noise as a function of frequency (power spectrum).
Note the peak at 0.2 Hz and decrease as a distant from coast.

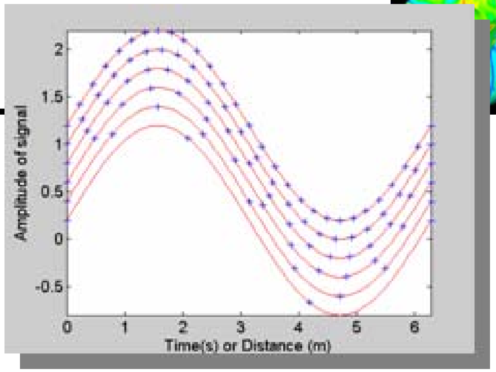
Instrument Filters



Sampling rate

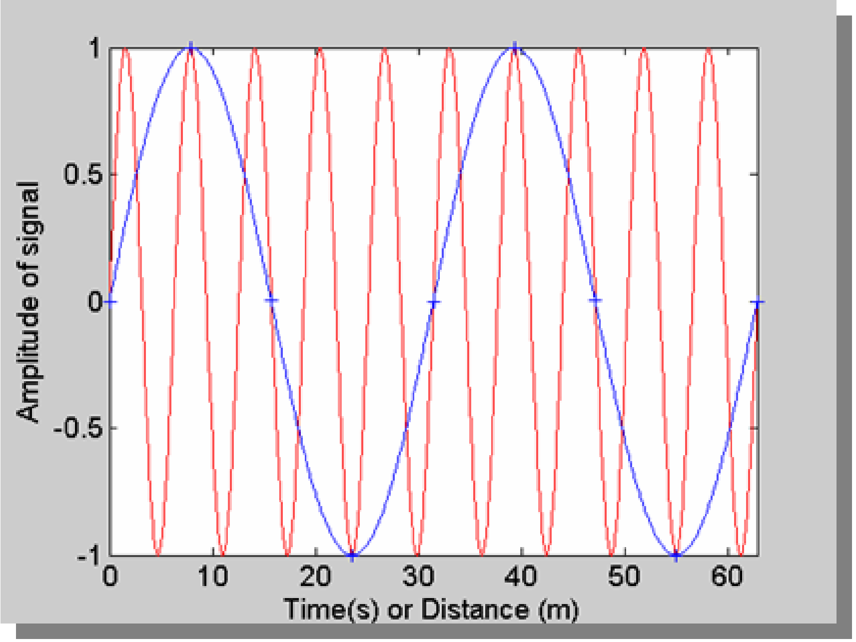


Sampling frequency, sampling rate is the number of sampling points per unit distance or unit time. Examples?



Nyquist Frequency (Wavenumber, Interval)

The frequency half of the sampling rate dt is called the Nyquist frequency $f_N=1/(2dt)$. The distortion of a physical signal higher than the Nyquist frequency is called aliasing.



The frequency of the physical signal is > f_N is sampled with (+) leading to the erroneous blue oscillation.

What happens in space? How can we avoid aliasing?

Discrete Convolution

Convolution is the mathematical description of the change of waveform shape after passage through a filter (system).

There is a special mathematical symbol for convolution (*):

$$y(t) = g(t) * f(t)$$

Here the impulse response function g is convolved with the input signal f. g is also named the „Green's function"

$$y_k = \sum_{i=0}^{m} g_i f_{k-i}$$

$$k = 0,1,2,\ldots,m+n$$

$$g_i \qquad i = 0,1,2,\ldots,m$$

$$f_j \qquad j = 0,1,2,\ldots,n$$

Convolution Example (Matlab)

```
>> x
x =
    0     0     1     0
>> y
y =
    1     2     1
>> conv(x,y)
ans =
    0     0     1     2     1     0
```

Impulse response

System input

System output

Convolution Example (pictorial)

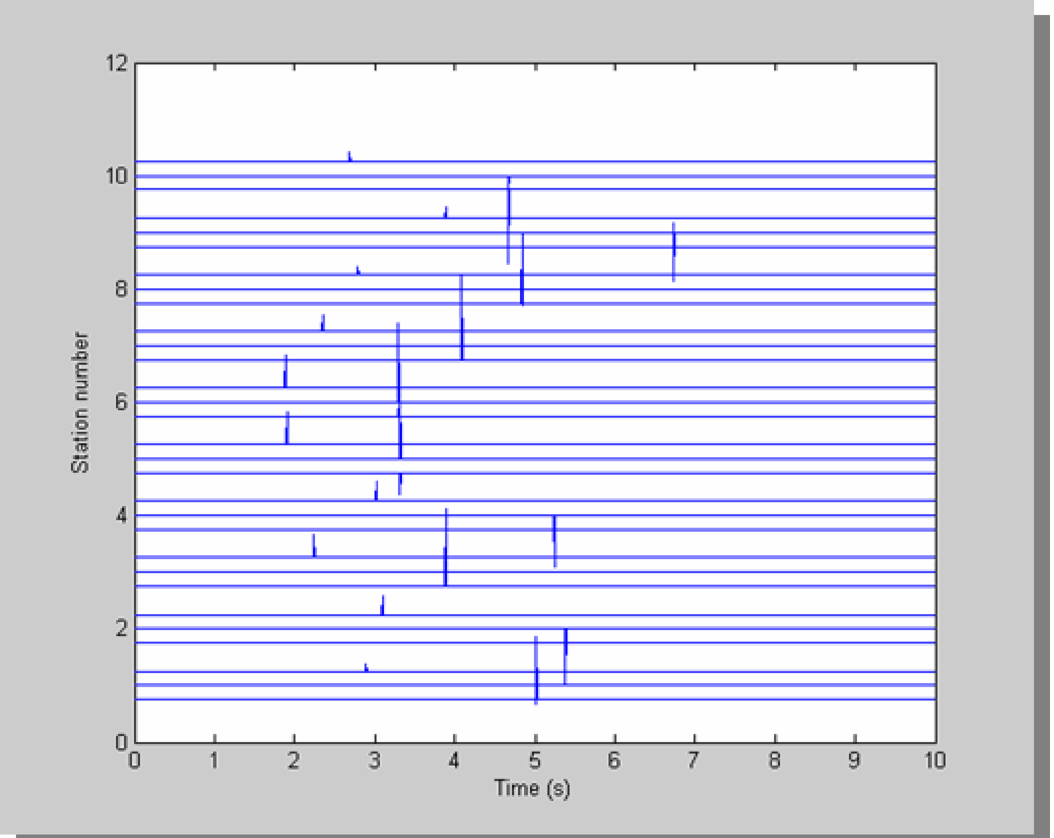
x „Faltung“ y x*y

								x*y
		0	1	0	0			0
					1	2	1	
		0	1	0	0			0
				1	2	1		
		0	1	0	0			1
			1	2	1			
		0	1	0	0			2
		1	2	1				
		0	1	0	0			1
	1	2	1					
		0	1	0	0			0
1	2	1						

Deconvolution

Deconvolution is the inverse operation to convolution.

When is deconvolution useful?



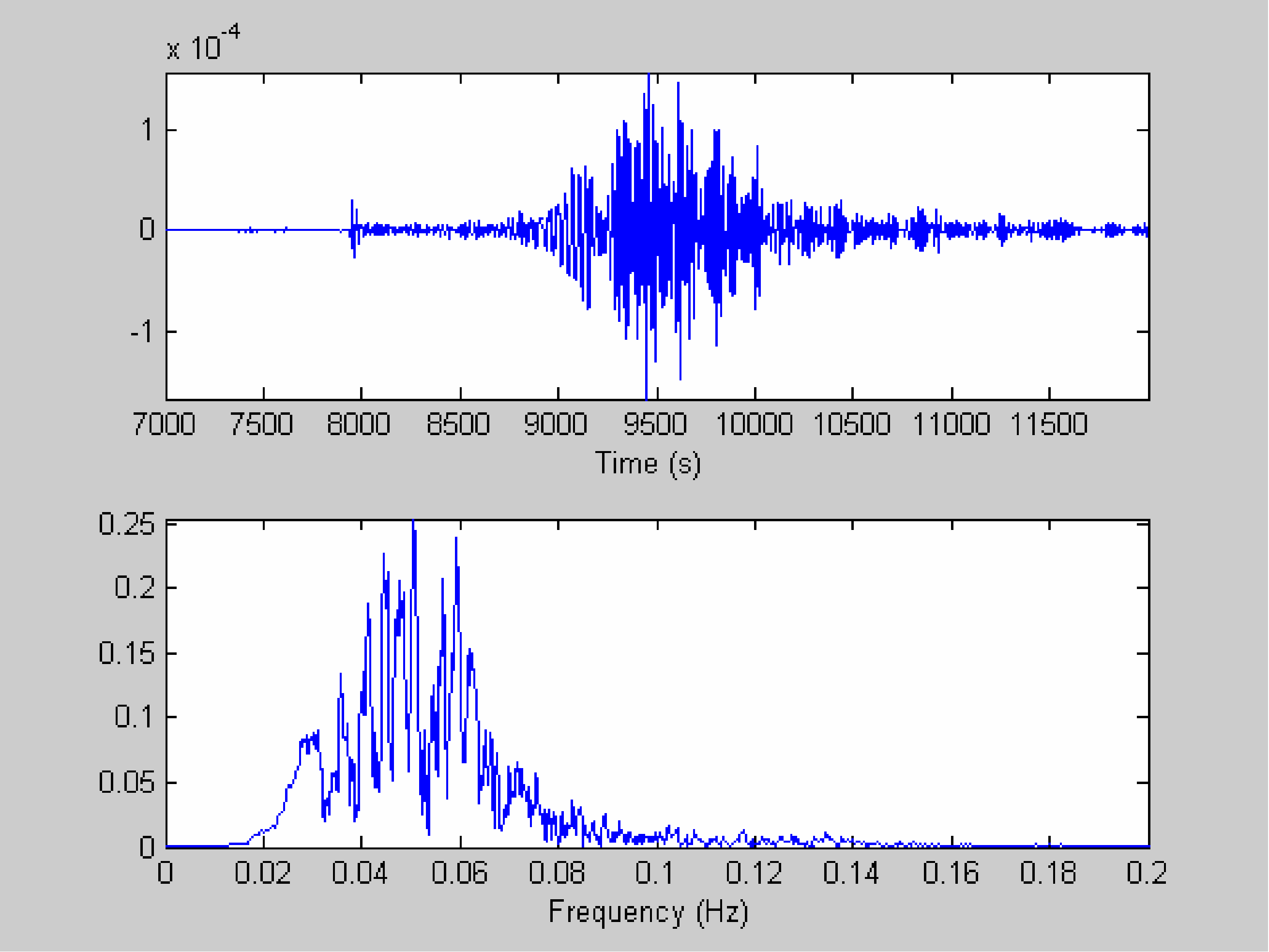
Digital Filtering

Often a recorded signal contains a lot of information that we are not interested in (noise). To get rid of this noise we can apply a filter in the frequency domain.

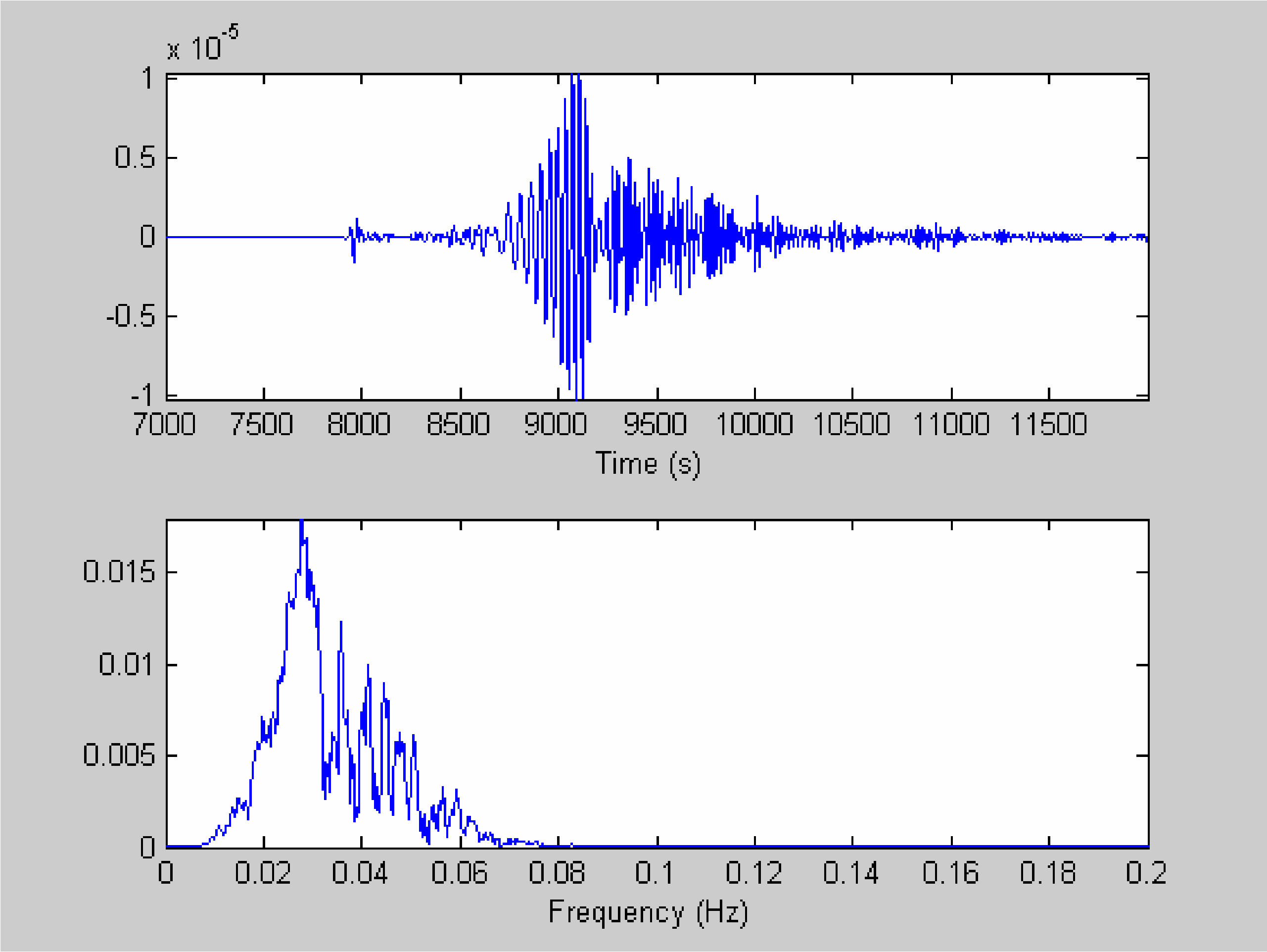
The most important filters are:

- High pass: cuts out low frequencies
- Low pass: cuts out high frequencies
- Band pass: cuts out both high and low frequencies and leaves a band of frequencies
- Band reject: cuts out certain frequency band and leaves all other frequencies

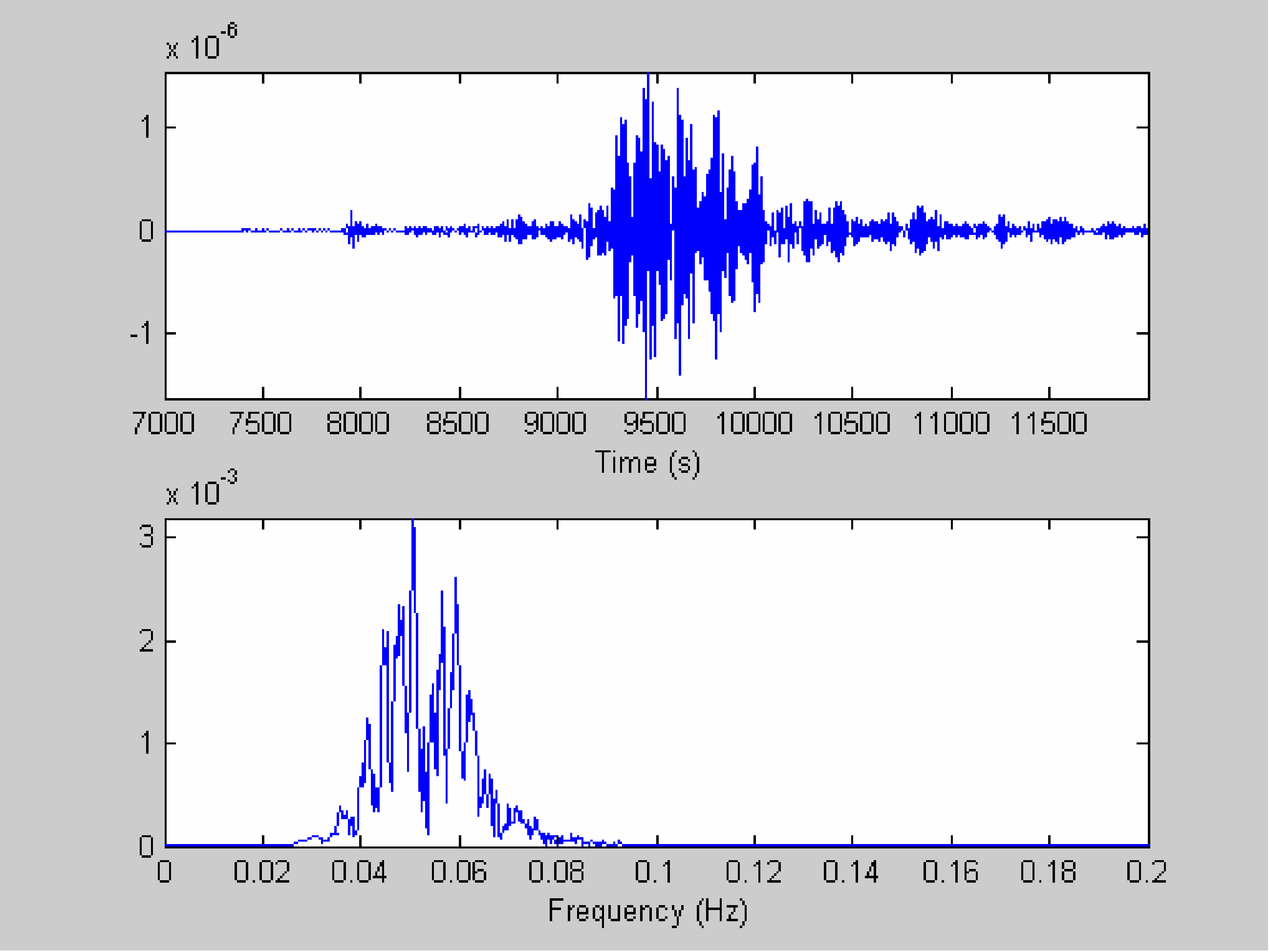
Digital Filtering



Low-pass filtering



High-pass filter



Band-pass filter