Geology 5660/6660: Applied Geophysics
Lecture 10

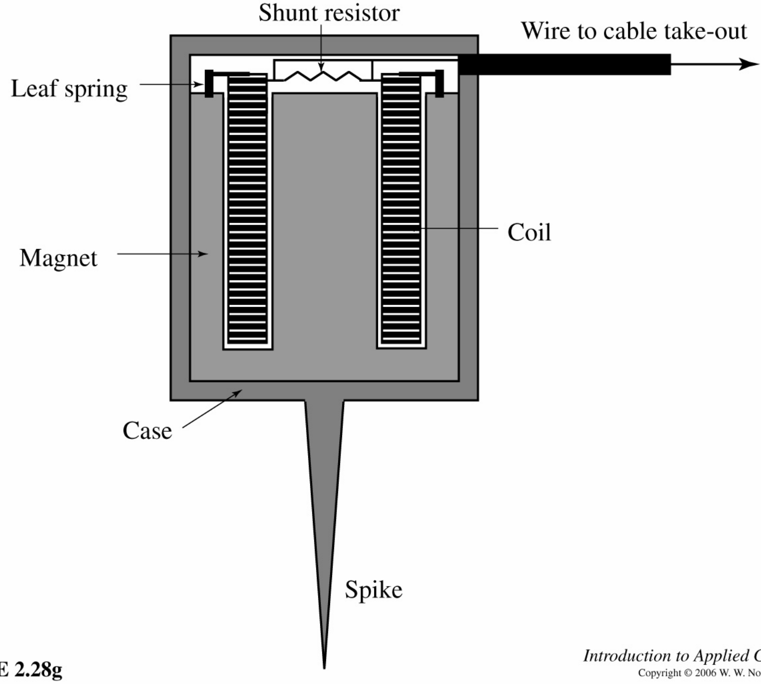


FIGURE 2.28g

Introduction to Applied Geophysics

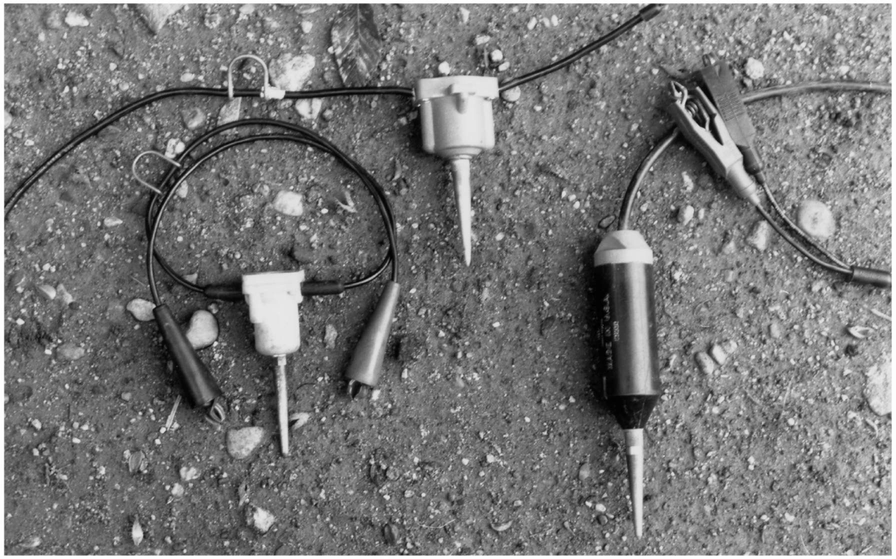


FIGURE 2.29g

Introduction to Applied Geophysics

Ravi Kanda, Spring 2020

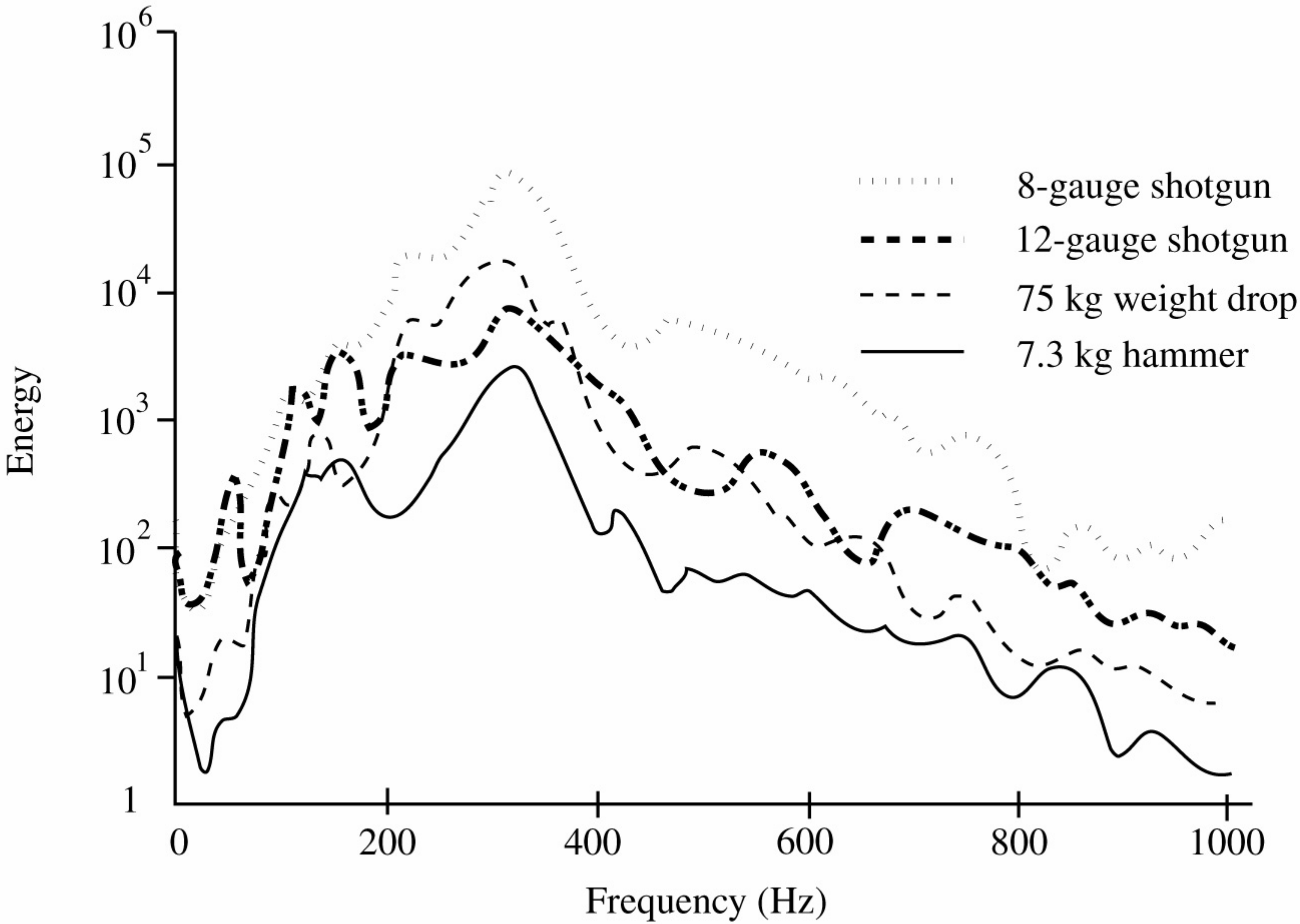


FIGURE 2.26g

Introduction to Applied Geophysics

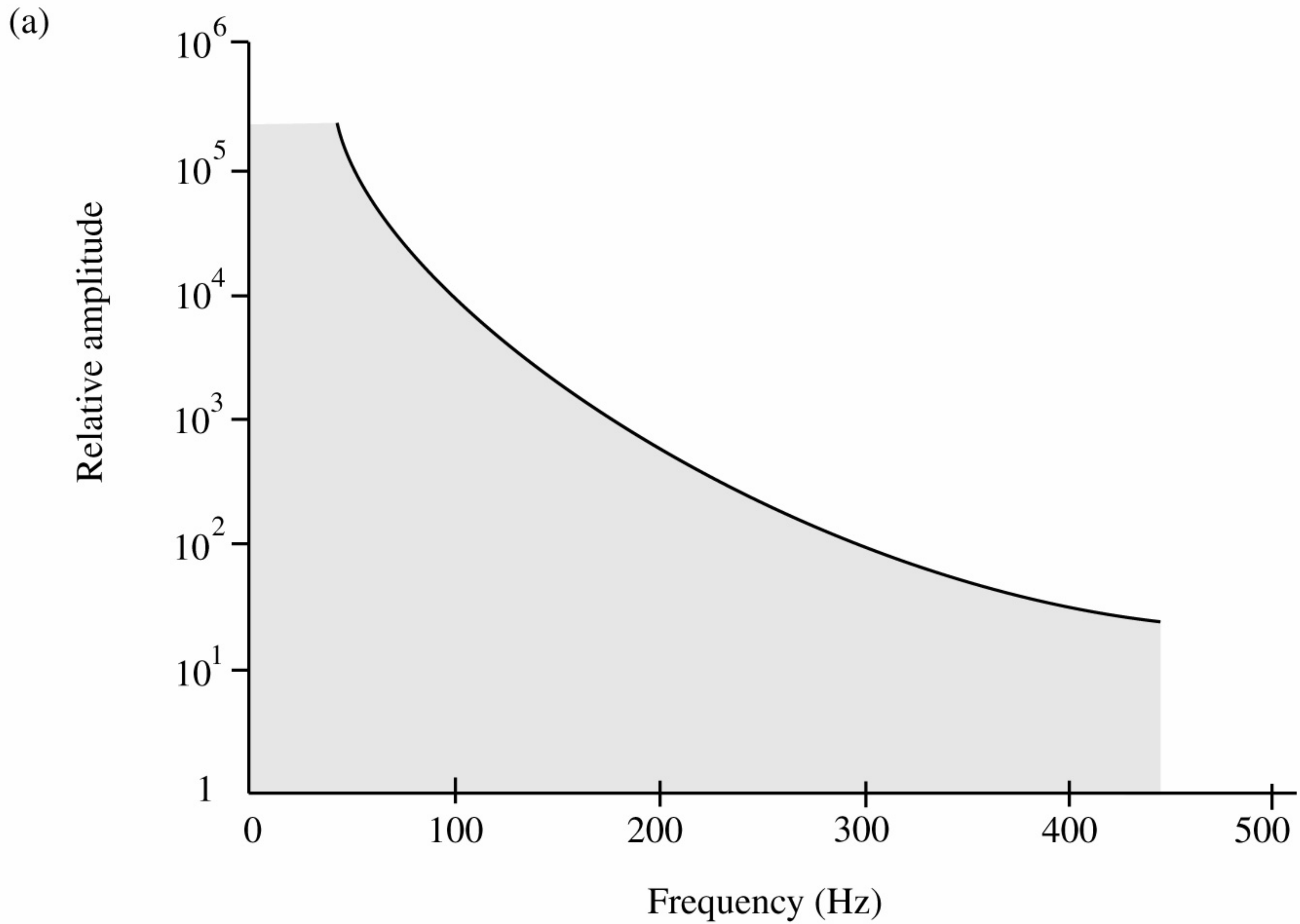


FIGURE 2.31g top

Introduction to Applied Geophysics

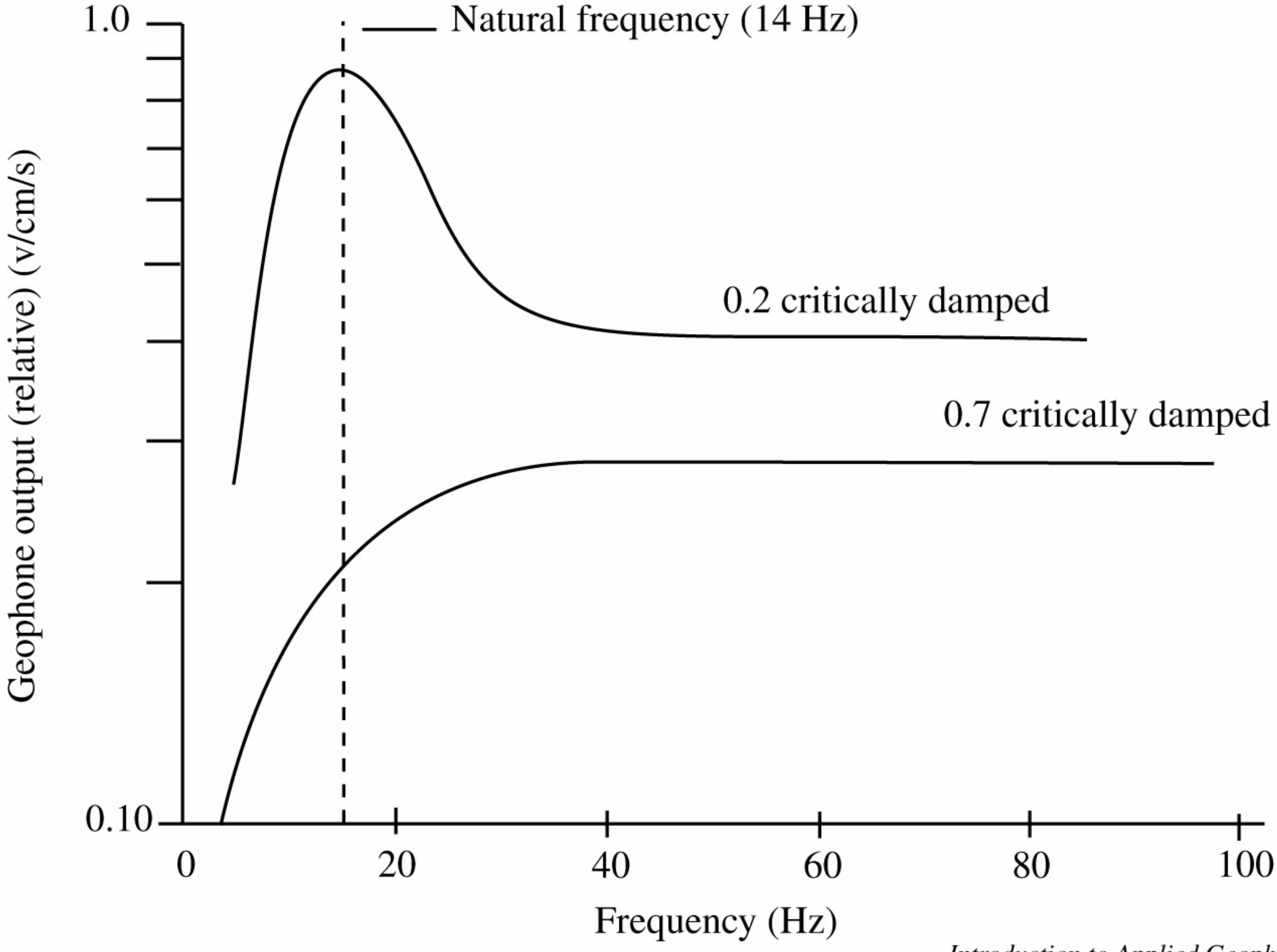


FIGURE 2.30g

Introduction to Applied Geophysics

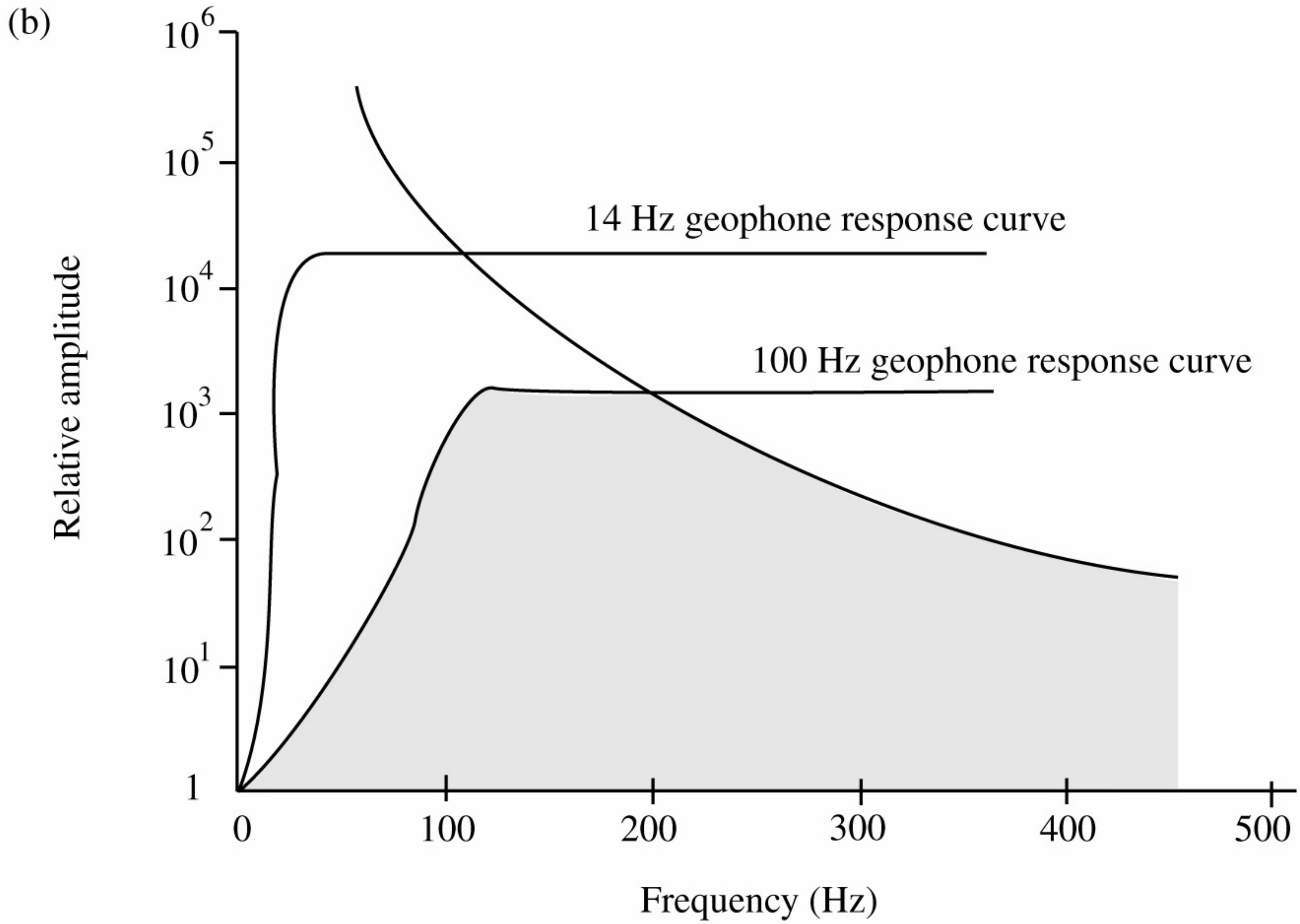


FIGURE 2.31g bottom

Introduction to Applied Geophysics

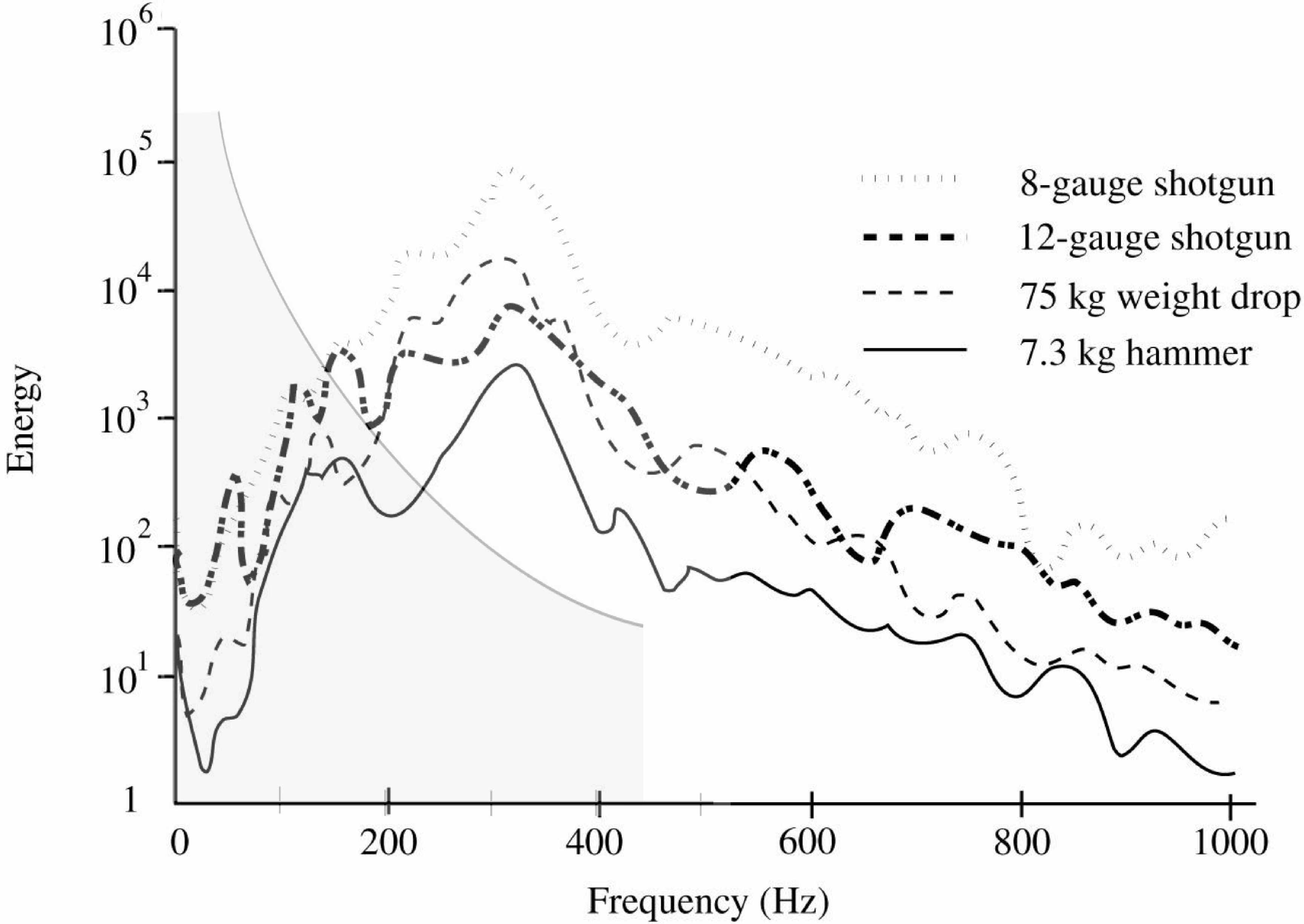


FIGURE 2.26g

Introduction to Applied Geophysics

Fourier Analysis and Synthesis

The great utility of the Fourier transform comes from its ability to decompose any function into a set of complex sinusoids. In the continuous case, the frequencies of the sinusoids range from -∞ to ∞ and have amplitudes and phases which are computed from the forward Fourier transform:

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-i\omega t} dt$$

This equation computes the complex coeficients, $H(\omega)$, of the complex sinusoids which, when summed (integrated), will yield $h(t)$. Usually $H(\omega)$ is decomposed into two separate real functions:

amplitude spectrum: $$A(\omega) = \left|H(\omega)\right| = \sqrt{\mathrm{Re}\left(H(\omega)\right)^2 + \mathrm{Im}\left(H(\omega)\right)^2}$$

phase spectrum: $$\phi(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left(H(\omega)\right)}{\mathrm{Re}\left(H(\omega)\right)}\right)$$

The inverse Fourier transform expresses the construction of $h(t)$ as a superposition of complex sinusoids:

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega) e^{i\omega t} d\omega$$

If we wish to use cyclical frequency, f, instead of angular frequency, ω, ($\omega = 2\pi f$) the Fourier transform pair is:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-2\pi i f t}\, dt$$

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{2\pi i f t}\, df$$

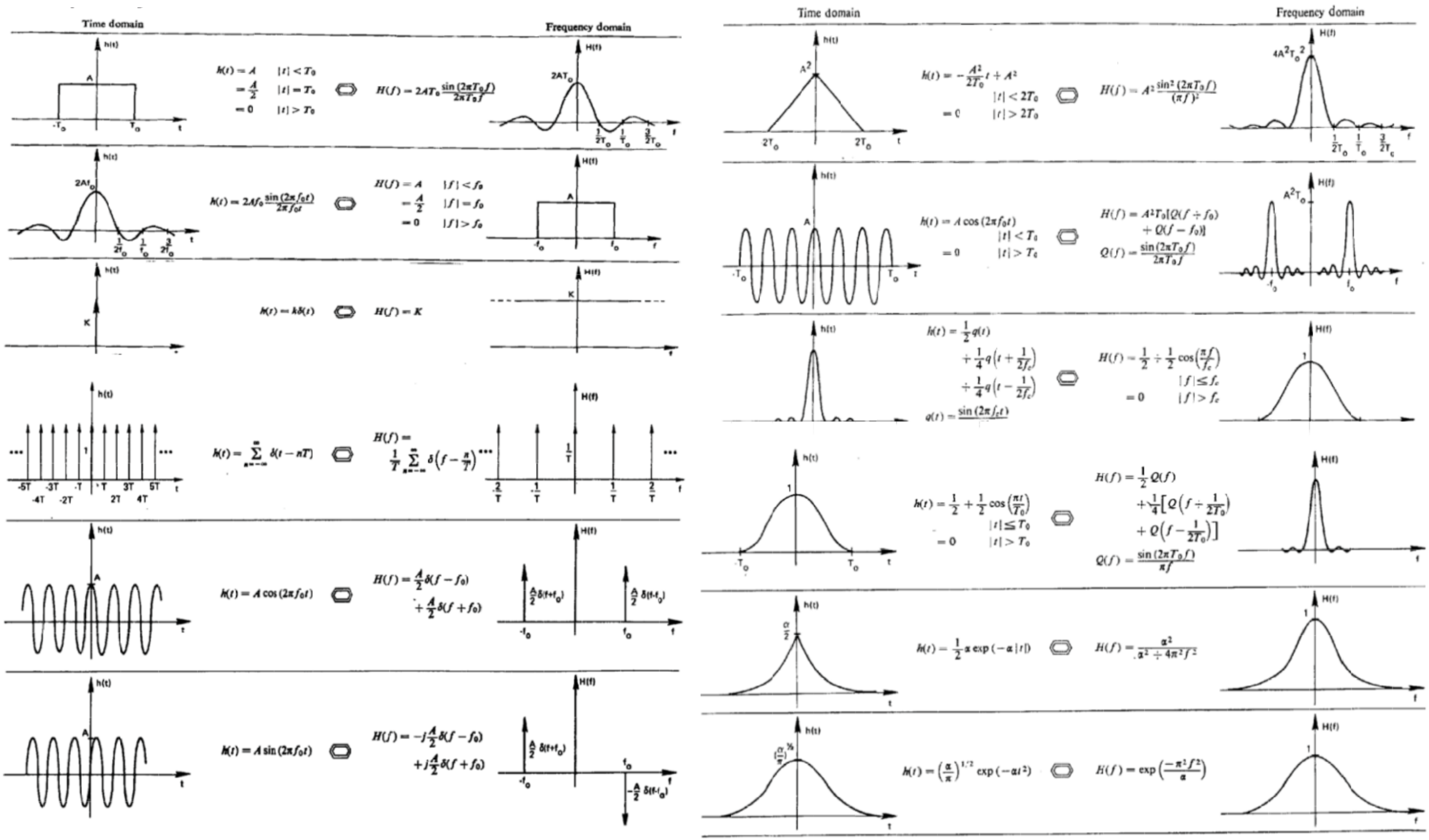
Methods of Seismic Data Processing – Margrave (Univ. Alb., 2006

Fourier Transform Pairs

The table below is reproduced from:

Brigham, E.O., 1974, The Fast Fourier Transform, Prentice Hall

Note: It is a remarkable fact that no signal can have finite length (i.e. compact support) in both the time and frequency domains.

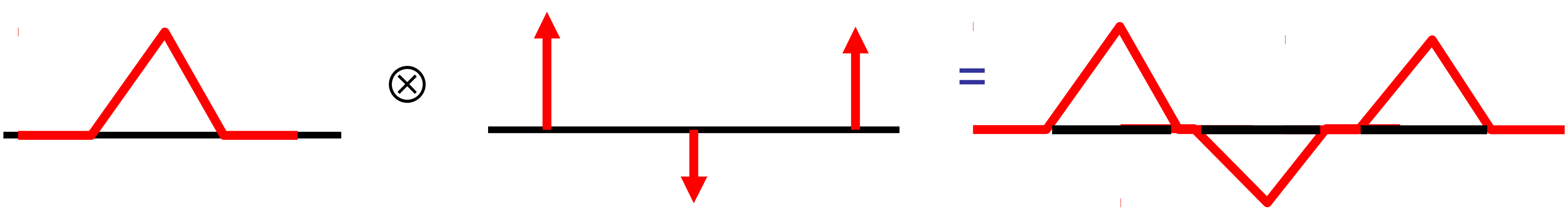


Signal recorded by a seismometer is a ***convolution*** of the wave source, the Earth response, and the seismometer response.

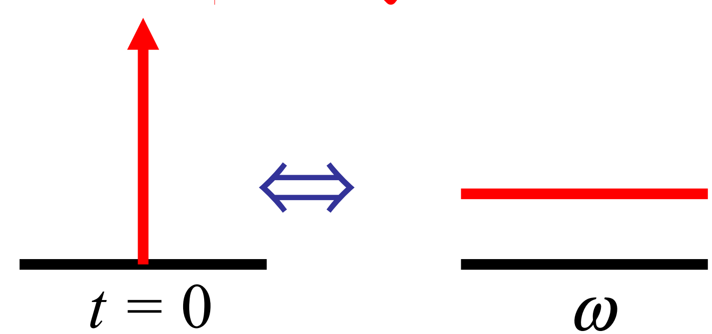
Wave Source ⊗ Earth Response ⊗ Seismometer Response

where ⊗ denotes convolution: $$(f \otimes g) = \int_{-\infty}^{\infty} f(t-x)\,g(x)\,dx$$

Example:



So, want seismometer response to look as much as possible like a single ***delta-function*** in time:



Convolution

We have seen that the convolution of discretely sampled vectors is written:

$$s_j = \sum_k r_k w_{j-k}$$

The analagous result for continuous functions is:

$$s(t) = \int_{-\infty}^{\infty} r(\tau) w(t-\tau) d\tau$$

We now show that the order of convolution is immaterial.
Let:

$$\tau' = t-\tau, \quad d\tau' = -d\tau, \quad \tau = t-\tau'$$

Then: $$s(t) = -\int_{\infty}^{-\infty} r(t-\tau') w(\tau') d\tau'$$

And: $$s(t) = \int_{-\infty}^{\infty} r(t-\tau') w(\tau') d\tau'$$

So: $$s = r \bullet w = w \bullet r$$

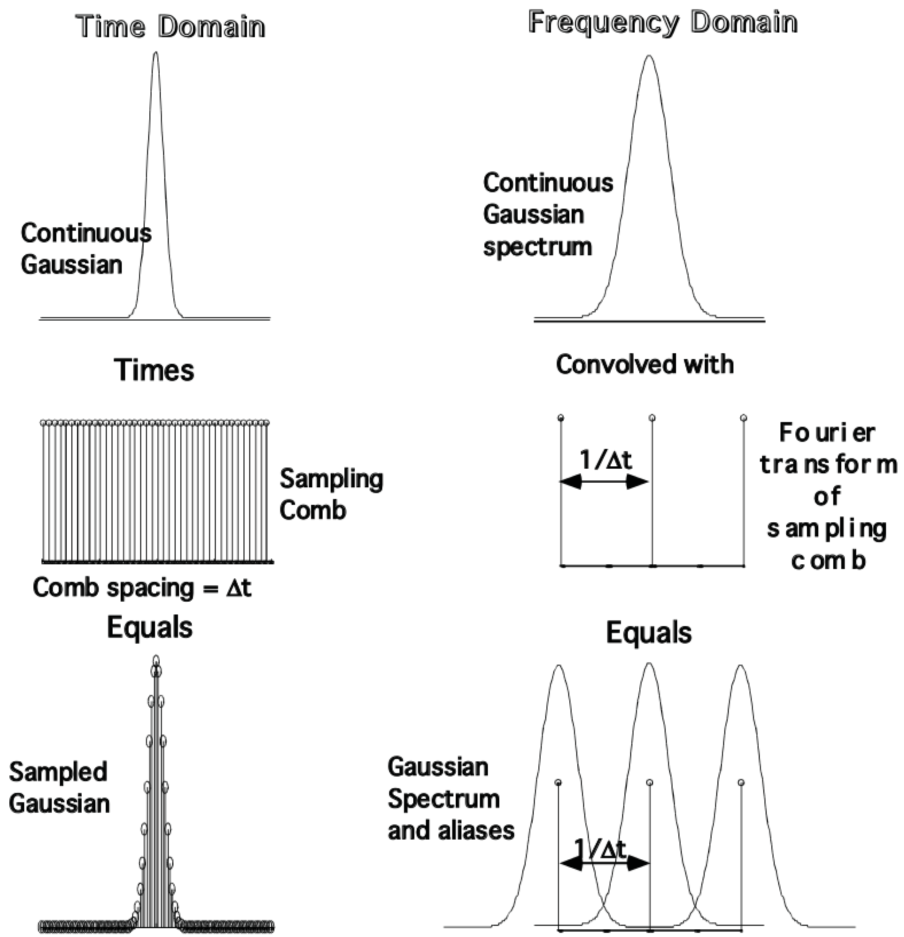
We also note that convolution is linear in the sense that:

$$(a+b) \bullet c = a \bullet c + b \bullet c$$

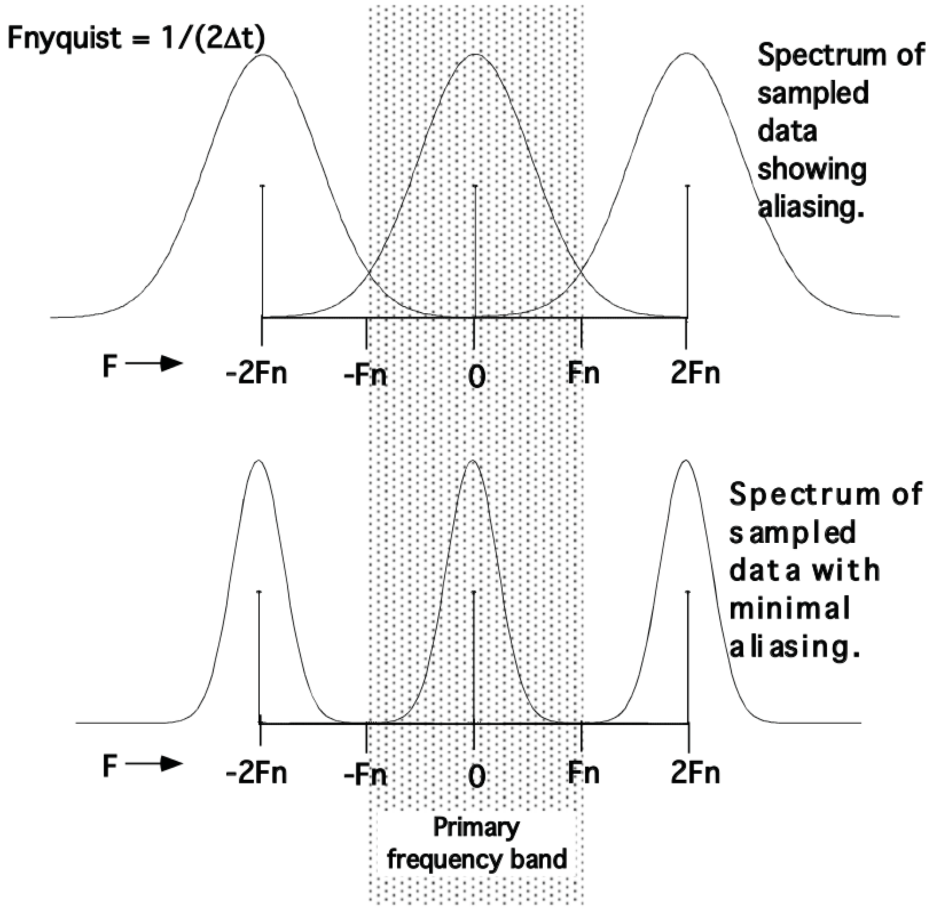
Methods of Seismic Data Processing – Margrave (Univ. Alb., 2006

Sampling

The analytic analysis of continuous signals is most useful for gaining a conceptual understanding of signal processing. In actual practice; however, the vast majority of work is done with discretly sampled functions. The process of sampling a continuous function in time can be viewed as a multiplication by a sampling comb.



So we have seen that sampling in the time domain causes the replication of the continuous spectrum in the frequency domain. The spacing between these spectral aliases is $1/\Delta t$ and it is customary to restrict our attention to the primary frequency band lieing between $-1/(2\Delta t)$ and $1/(2\Delta t)$. The frequency $F_n = 1/(2\Delta t)$ is called the Nyquist frequency and is the limiting frequency of the sampled data.



Methods of Seismic Data Processing – Margrave (Univ. Alb., 2006