Geology 5660/6660: Applied Geophysics
Lecture 09

Topics covered so far (& today):

Seismic Amplitude depends on

Source amplitude

Geometrical Spreading (spread of energy in increasing volume).

Uniform halfspace *spherical spreading*: $A = \frac{A_0}{r}$

For refracted *head wave*, *cylindrical spreading*: $A = \frac{A_0}{\sqrt{r}}$

Anelastic Attenuation depends on *Quality Factor* *Q*: $A = A_0 e^{-\frac{\pi f x}{QV}}$

Mode Partitioning at interfaces (reflection, refraction):

Energy density: J/m^3

$$\hat{u} = \mu k^2 A^2/4 = \rho\omega^2 A^2/4 = k \Rightarrow \hat{e} = \rho\omega^2 A^2/2$$

Energy flux density: $J/s/m$ or W/m

$$\hat{e}.\Delta S = \rho \Delta S \omega^2 A^2/2, \quad \text{where, } \Delta S = \text{Surface area}$$

Energy flux: $J/s/m^2$ or W/m^2

$$\hat{e}.c = \rho c \omega^2 A^2/2, \quad \text{where, } c = (\alpha \text{ or } \beta)$$

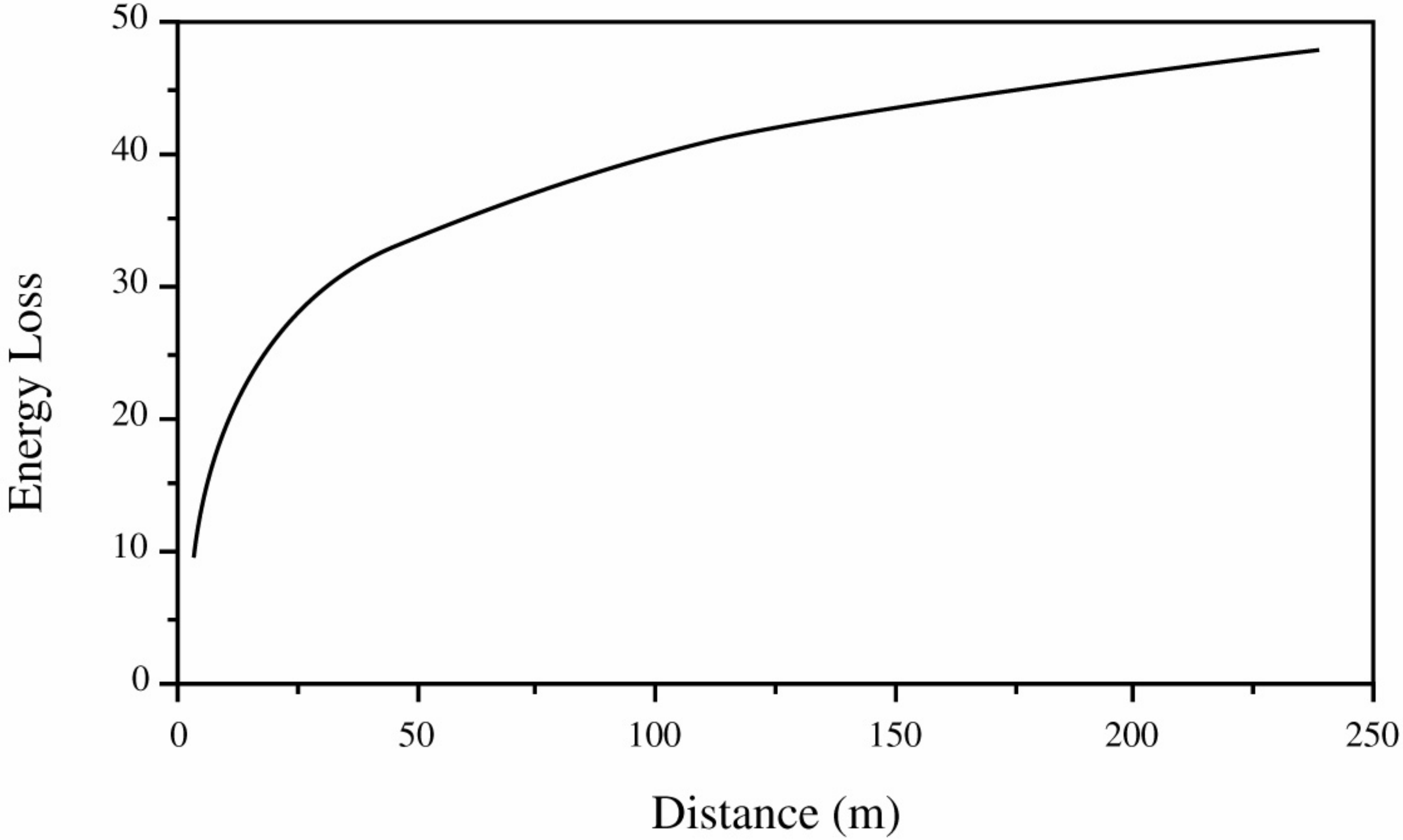
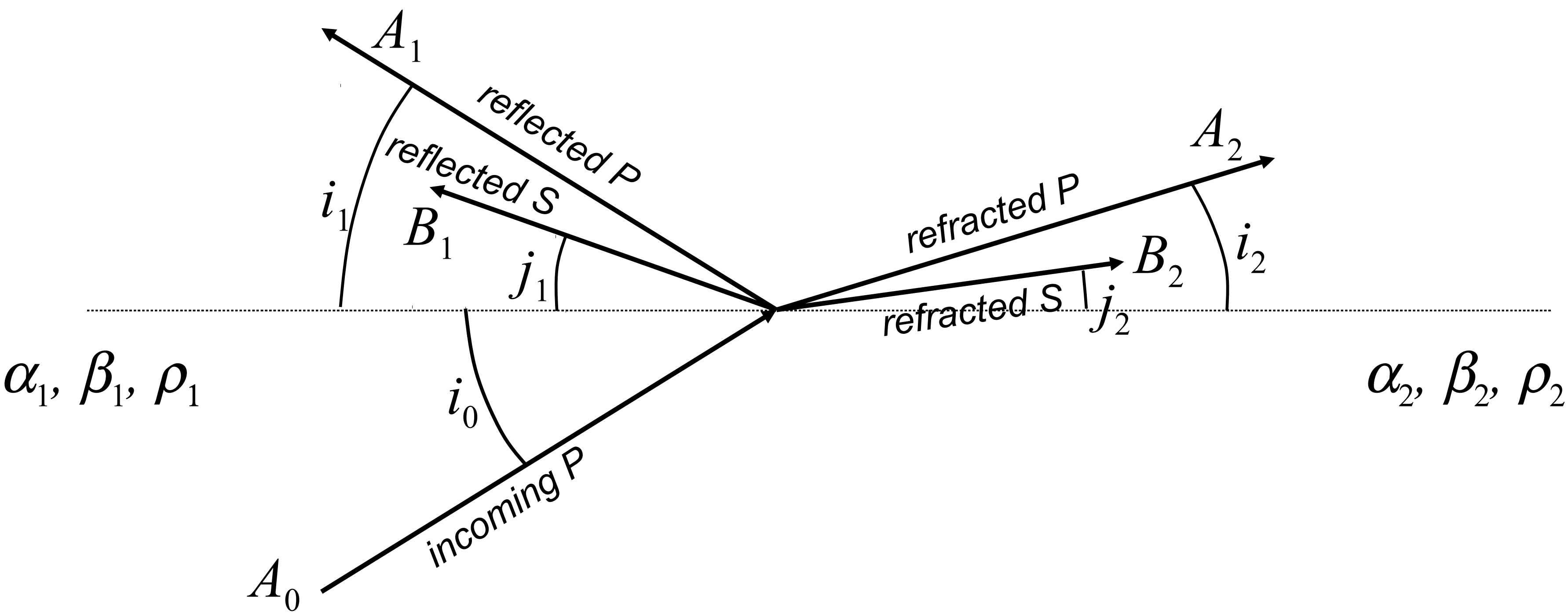


FIGURE 2.23g

Mode Conversions:

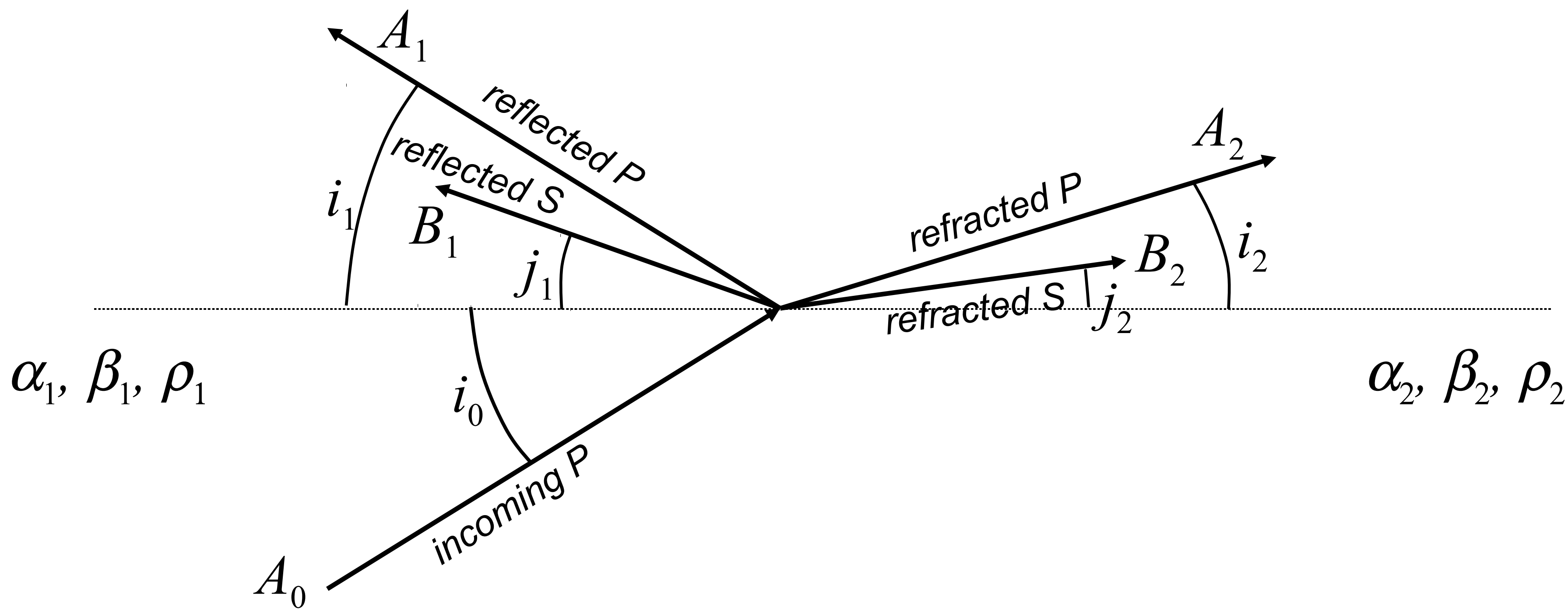


Snell's Law predicts ray angles: For $i_0 \leq i_c$, where for example $i_c = \sin^{-1}\left(\frac{\alpha_1}{\alpha_2}\right)$

$$\frac{\sin i_0}{\alpha_1} = \frac{\sin i_1}{\alpha_1} = \frac{\sin j_1}{\beta_1} = \frac{\sin i_2}{\alpha_2} = \frac{\sin j_2}{\beta_2}$$

However Snell's law gives no information about *amplitudes*!

Mode Conversions:



Consider particle DISPLACEMENTS & TRACTIONS at the interface:

(a) u_1, u_3 are nonzero ($u_2 = 0$) and must be continuous across the boundary:

$$u_1^+ = u_1^- \qquad u_3^+ = u_3^-$$

(b) Tractions must also be continuous, which leads to:

$$\sigma_{13}^+ = \sigma_{13}^- \qquad \sigma_{11}^+ = \sigma_{11}^-$$

(c) Finally, energy must be conserved:
Sum of energy in reflected/refracted wave amplitudes = incident wave energy.

$$u_1^+ = u_1^- \Rightarrow A_0 + A_1 + B_1 \cot j_1 = A_2 - B_2 \qquad (1)$$

$$u_3^+ = u_3^- \Rightarrow \left(A_0 - A_1\right)\cot i_0 - B_1 = A_2 \cot i_2 - B_2 \qquad (2)$$

$$\sigma_{13}^+ = \sigma_{13}^- \Rightarrow \rho_1 \beta_1^2 \left[\left(1 - \cot^2 i_1\right)\left(A_0 + A_1\right) - 2B_1 \cot j_1\right]$$

$$= \rho_2 \beta_2^2 \left[\left(1 - \cot^2 i_2\right) A_2 - 2B_2 \cot j_2\right] \qquad (3)$$

$$\sigma_{11}^+ = \sigma_{11}^- \Rightarrow \rho_1 \beta_1^2 \left[\left(A_0 - A_1\right)\cot i_1 + \left(\cot^2 j_1 - 1\right) B_1\right]$$

$$= \rho_2 \beta_2^2 \left[2A_2 \cot i_2 + \left(\cot^2 j_2 - 1\right) B_2\right] \qquad (4)$$

These four equations in five unknowns (A_0, A_1, B_1, A_2, B_2) are called the *Zoeppritz' Equations*. If we fix the amplitude for the incident wave (e.g., $A_0 = 1$), we can solve for the other four.

Reflected & refracted amplitudes depend on Impedence Contrast, which is a function of energy partitioning at the boundary. Recall that
Energy flux ($J/s/m^2$ or W/m^2), $\hat{e}.c = \rho c \omega^2 A^2/2$, where, $c = (\alpha$ or $\beta)$

So, as the wave moves from layer 1 to 2 (ignoring scattering):

$$\rho_1 c_1 \omega^2 A_1^{\,2}/2 = \rho_2 c_2 \omega^2 A_2^{\,2}/2$$

$$\Rightarrow \quad A_2/A_1 = \sqrt{\{(\rho_1 c_1)/(\rho_2 c_2)\}} = \sqrt{(Z_1/Z_2)}$$

A normally-incident ($\theta = 0$) P-/S-wave with amplitude A_i produces a ***reflected P/S*** with amplitude ($V = \alpha$ or β):

$$\frac{A_{rfl}}{A_i} = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1} \equiv \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (\textit{reflection coefficient})$$

and a ***refracted P/S***:

$$\frac{A_{rfr}}{A_i} = \frac{2\rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1} \equiv \frac{2Z_1}{Z_2 + Z_1} \quad (\textit{refraction coefficient})$$

where $Z_i = \rho_i V_i$ is the *impedance* in layer *i*. *These amplitude ratios are sometimes "energy normalized" by multiplying with the ratio* $\sqrt{(Z_2/Z_1)}$

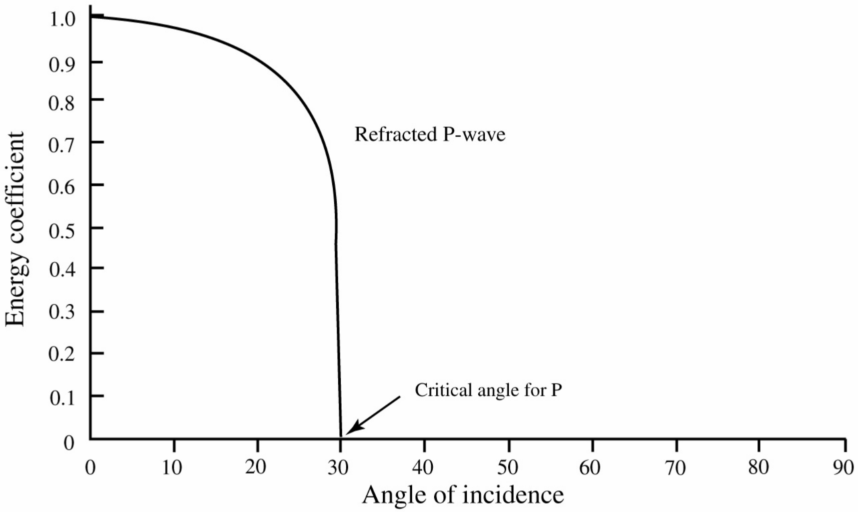


FIGURE 2.25g top left

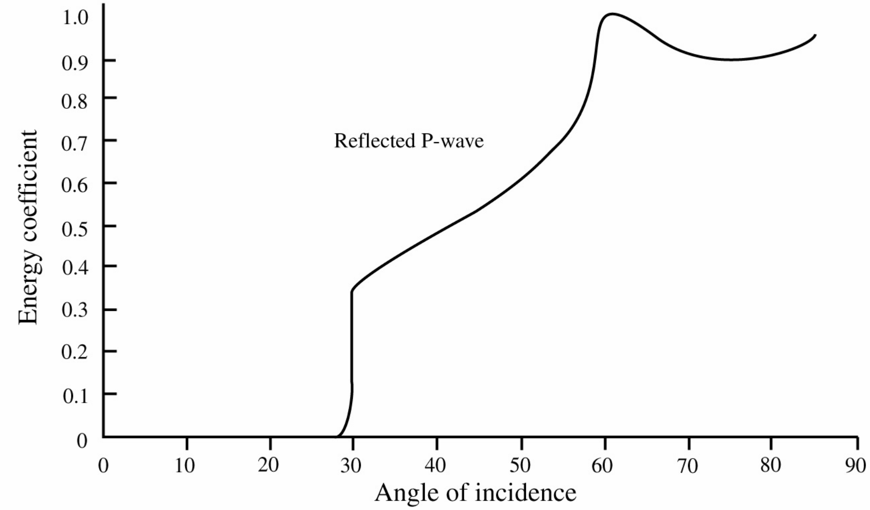


FIGURE 2.25g top right

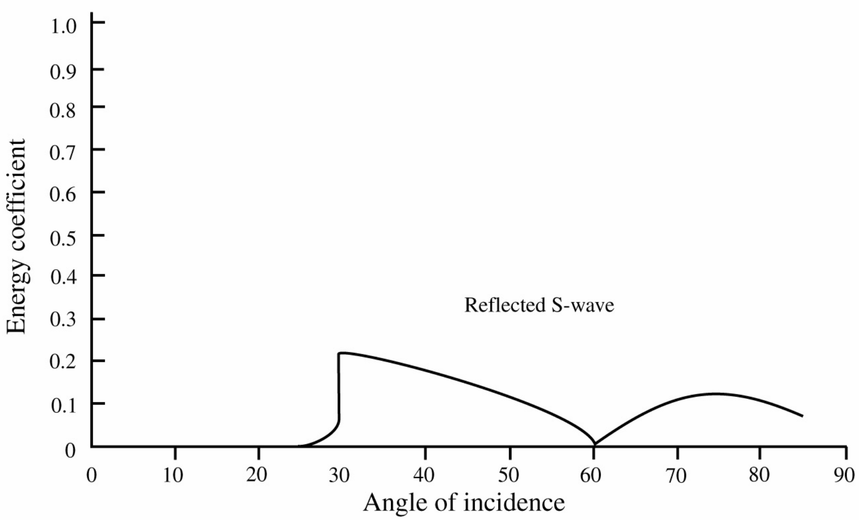


FIGURE 2.25g bottom left

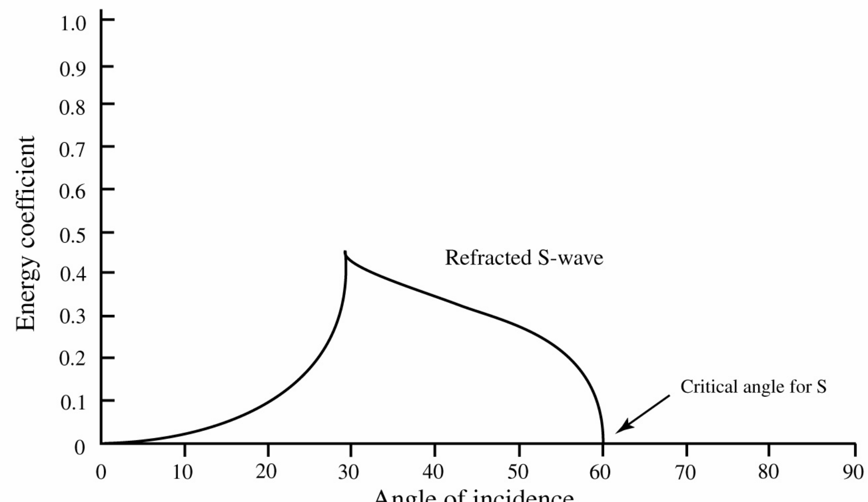
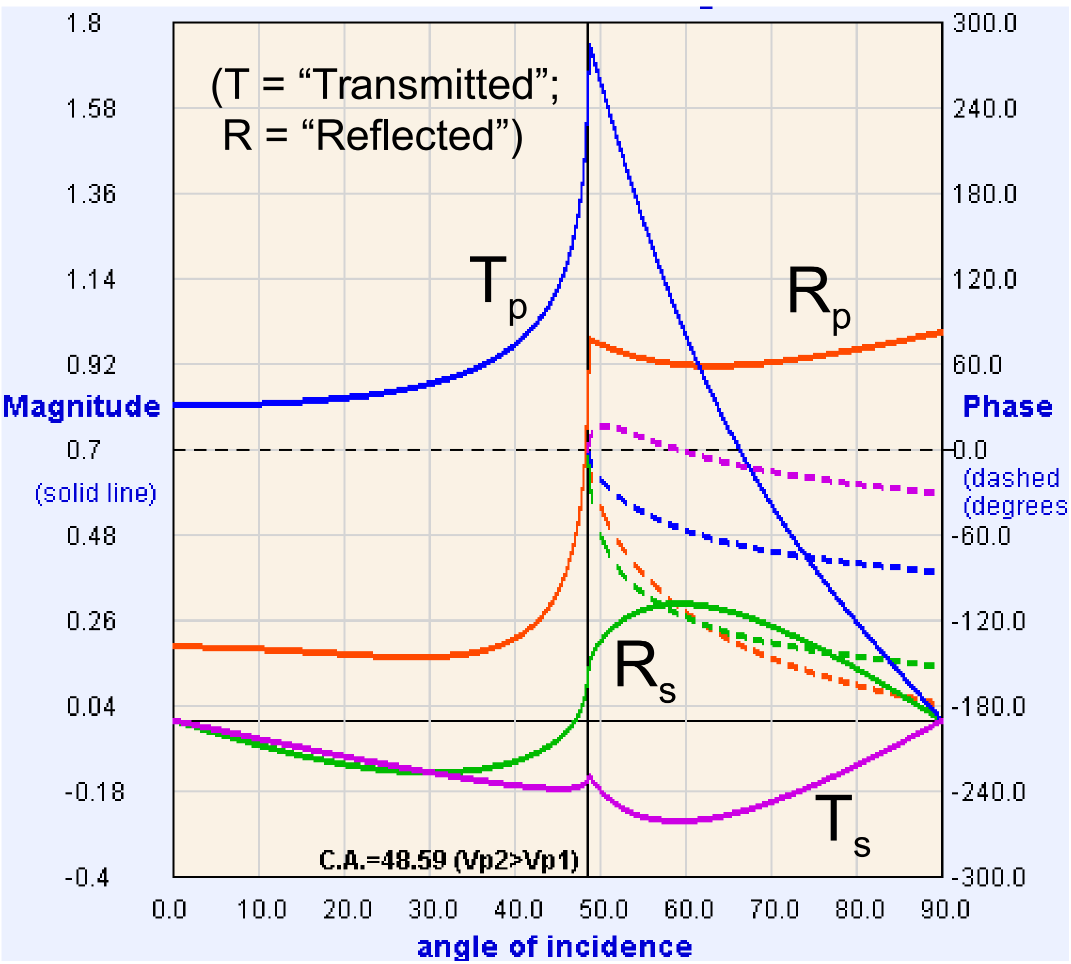


FIGURE 2.25g bottom right



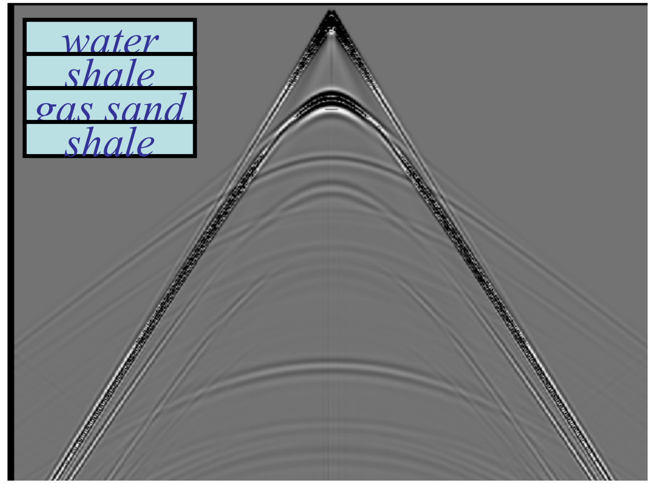
Crewes Zoeppritz Explorer

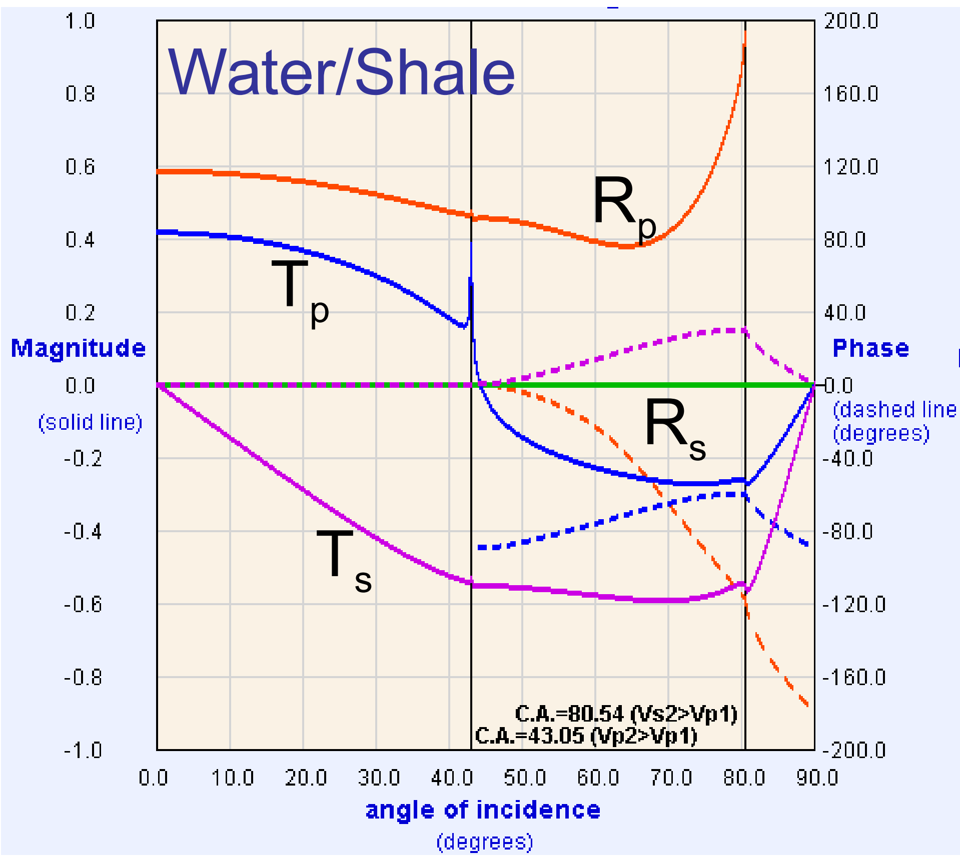
Model from: http://www.crewes.org/ResearchLinks/ExplorerPrograms/ZE/index.html

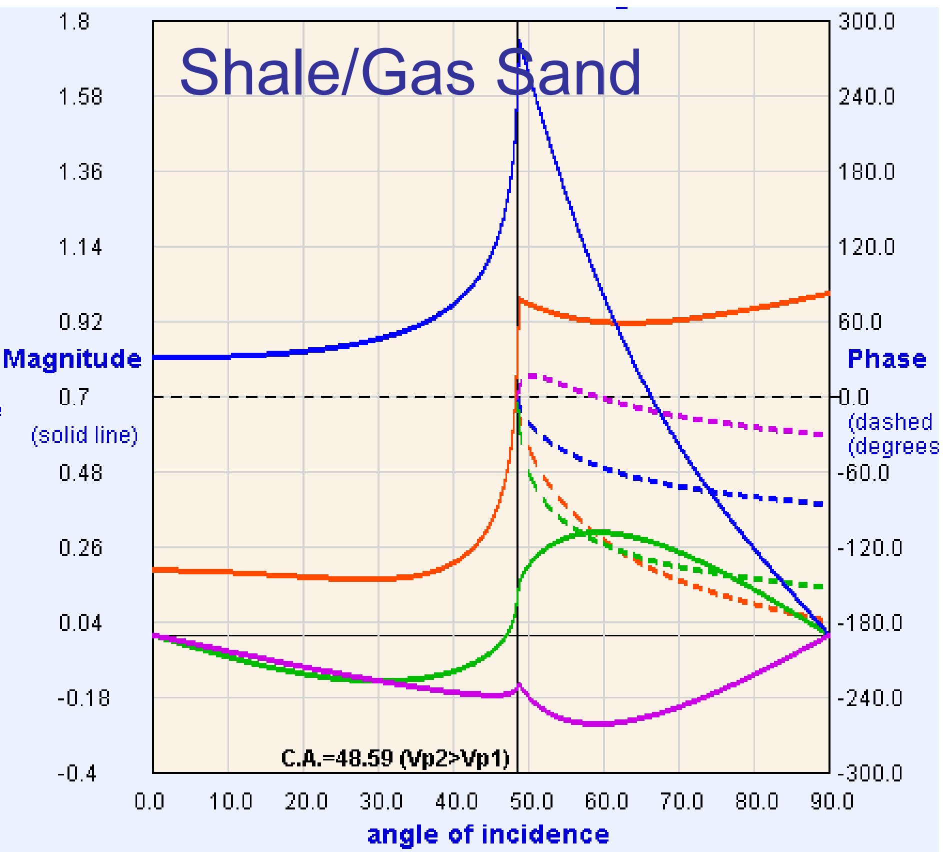
$\rho_1 = 2000,\ \rho_2 = 2200\ \ (kg/m^3)$

$\alpha_1 = 3000,\ \alpha_2 = 4000\ \ (\mathrm{m/s})$

$\beta_1 = 1500,\ \beta_2 = 2000\ \ (\mathrm{m/s})$


water
shale
gas sand
shale


Water/Shale
Rp
Tp
Rs
Ts
Magnitude
(solid line)
Phase
(dashed line
(degrees)
C.A.=80.54 (Vs2>Vp1)
C.A.=43.05 (Vp2>Vp1)
angle of incidence
(degrees)
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
200.0
160.0
120.0
80.0
40.0
0.0
-40.0
-80.0
-120.0
-160.0
-200.0
0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0


Shale/Gas Sand
Magnitude
(solid line)
Phase
(dashed
(degrees
C.A.=48.59 (Vp2>Vp1)
angle of incidence
1.8
1.58
1.36
1.14
0.92
0.7
0.48
0.26
0.04
-0.18
-0.4
300.0
240.0
180.0
120.0
60.0
0.0
-60.0
-120.0
-180.0
-240.0
-300.0
0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0