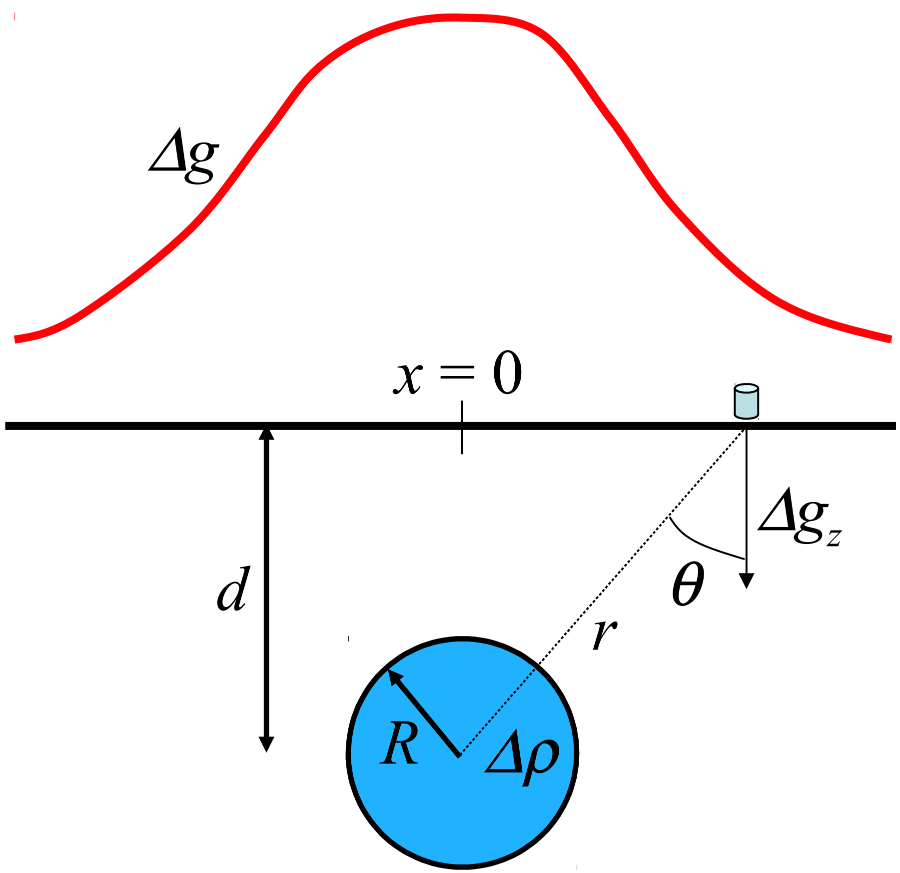
Gravity Anomaly: Spherical Source

Recall gravitational acceleration from a spherical source is given by: $\Delta\vec{g} = \hat{r}\dfrac{G\Delta m}{r^2}$

We measure Δg along a ~flat (ground) surface (i.e., x varies but d is fixed). So,



$$\Delta\vec{g} = \hat{r}\frac{G\Delta m}{x^2+d^2} = \hat{r}\frac{4\pi G R^3 \Delta\rho}{3\left(x^2+d^2\right)}$$

Thus,

$$\Delta g_z = \frac{4\pi G R^3 \Delta\rho}{3\left(x^2+d^2\right)}\cos\theta = \frac{4\pi G R^3 \Delta\rho d}{3\left(x^2+d^2\right)^{\frac{3}{2}}}$$

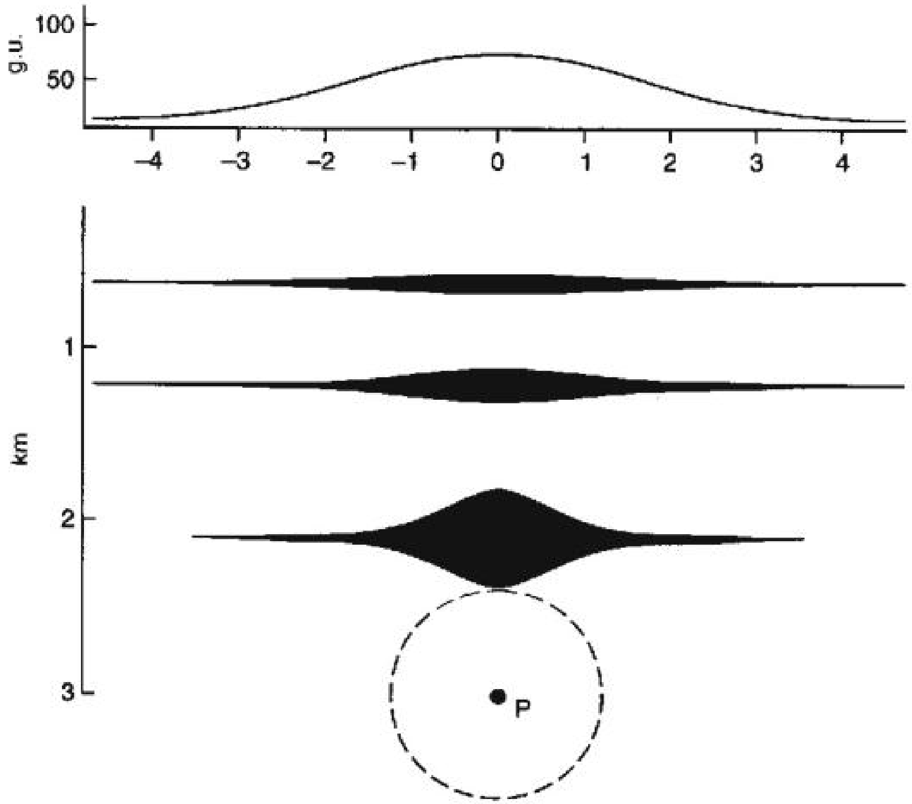
Diapir: For a 2 km diameter sphere (r = 1 km) buried 2 km deep (d), and having a density contrast $\Delta\rho$ of 1000 kg/m^3:

$\Delta g_{z,max}$ ~ 7 m-Gal, or 7000 μ-Gal

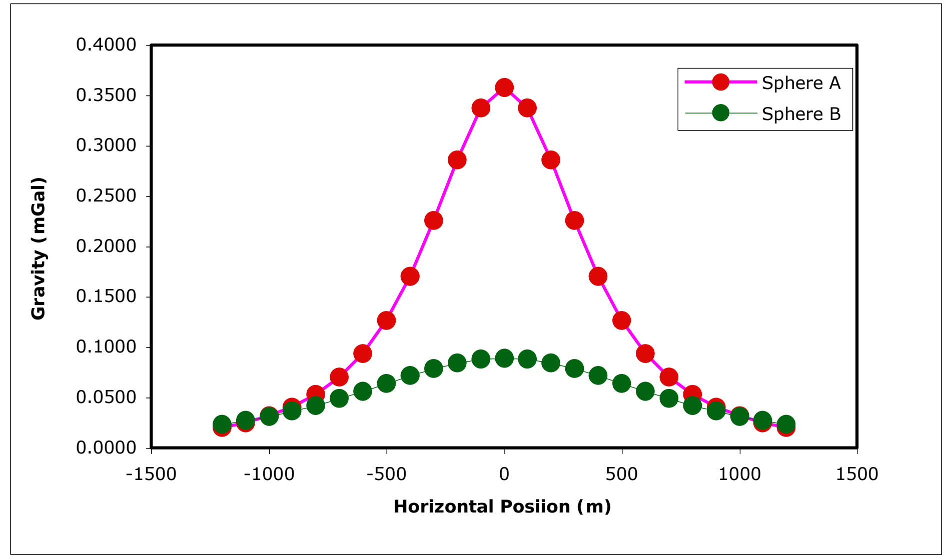
This is small relative to many gravity corrections discussed in the last module, which is why we need to estimate the residual anomaly, Δg_i accurately & precisely.

Gravity Anomaly: Spherical Source

NOTE: Given only the gravity anomaly, the source of gravity is ***non-unique***!



Saltus & Blakely, GSA Today 2011



We would estimate the same anomaly, if the sphere:

Had Higher density contrast, $\Delta\rho$
+ smaller radius, R

OR

Were at greater depth, d
+ had larger radius, R

However gravity unfairly gets a bad rap: ALL geophysical (& ALL geological) models are non-unique. Gravity narrows the *solution space* (the range of possible solutions).

Gravity Anomaly: Infinite slender "rod"

For a horizontal cylindrical source extending in-and-out of this page, we again measure Δg along a ~flat (ground) surface (i.e., x varies but d is fixed). However, we first start with the anomaly associated with an infinite slender "rod", also extending in-and-out of the plane of this page, with a mass per unit length $\boldsymbol{\lambda}$, and at a depth, $\boldsymbol{d}$.

Mass of differential element, $dm = \lambda\, dy$
so that

$$d(\delta g_{rod}) = \frac{G dm}{r^2}\cos\theta = \frac{G(\lambda\, dy)}{[y^2+(x^2+d^2)]}\left(\frac{d}{\sqrt{[y^2+(x^2+d^2)]}}\right) = G\lambda d\left(\frac{dy}{\sqrt{[y^2+(x^2+d^2)]^3}}\right) \qquad (1)$$

So,

$$\delta g_{rod} = G\lambda d\int_{-\infty}^{\infty}\left(\frac{dy}{\sqrt{[y^2+(x^2+d^2)]^3}}\right); \quad \textit{Substitute } y=(x^2+d^2)\tan(\theta) \Rightarrow dy=(x^2+d^2)\sec^2(\theta)\,d\theta$$

$$= \frac{G\lambda d}{(x^2+d^2)}\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(\theta)\,d\theta$$

$$= \frac{G\lambda d}{(x^2+d^2)}\langle\sin(\theta)\rangle_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = \frac{2G\lambda d}{(x^2+d^2)}$$

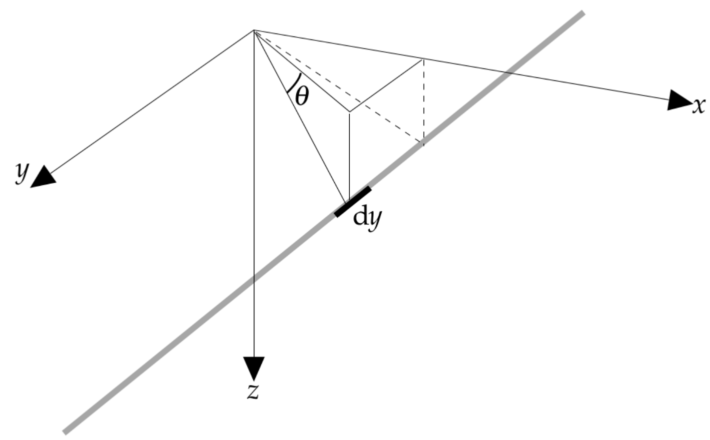
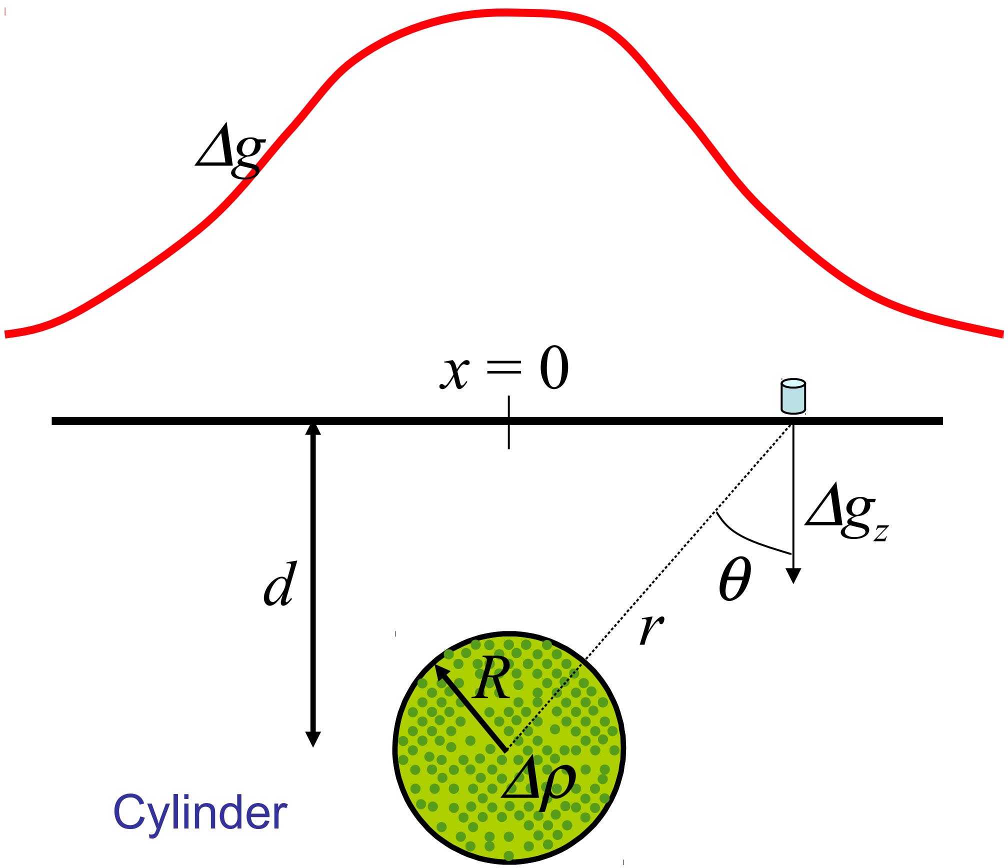


Fig. 2.51 Geometry for calculation of the gravity anomaly of an infinitely long linear mass distribution with mass *m* per unit length extending horizontally along the *y*-axis at depth *z*.

Lowrie, 2007

Gravity Anomaly: Horizontal Cylindrical Source

Now, for a horizontal cylindrical source extending in-and-out of this page, we assume that distances to the ~flat (ground) surface (i.e., x and d) are much larger than the diameter of the cylinder, R. So, $(x,d) \gg R$.



Under these conditions, we can fill the cylinder's cross-section (below left) with infinitely many slender "rods", in which case, the total mass per unit length for the circular cross-section, $\boldsymbol{\lambda} = \Delta\rho\,(\pi R^2)$, and:

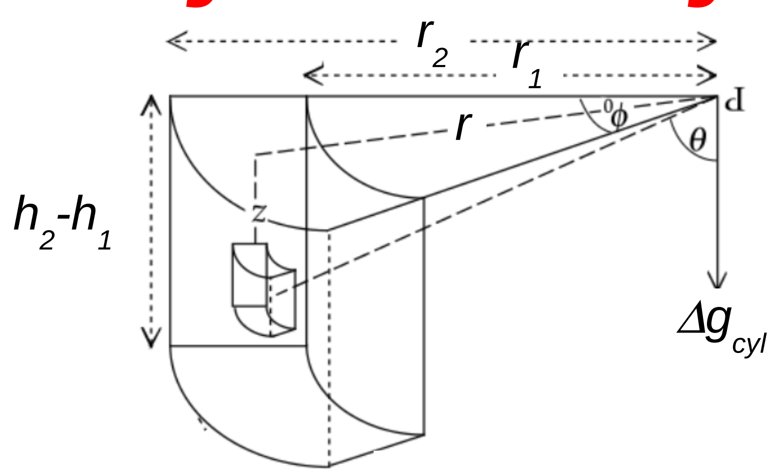
$$\Delta g_z = 2\pi G R^2 \Delta\rho \frac{d}{\left(x^2 + d^2\right)}$$

Syncline: For a 2 km diameter "cylindrical" axial core (r = 1 km) located at 2 km depth (d), and having a density contrast $\Delta\rho$ of 1000 kg/m^3:

$\Delta g_{z,max}$ ~ 21 m-Gal, or 21,000 μ-Gal

This is 3X larger than for a similarly located spherical body with similar density variation, because the effect of the "infinitely" long cylinder – the anomaly also decays slower than the more spatially restricted sphere !

Gravity Anomaly: Vertical Cylindrical Source



As with Terrain Correction, we proceed as follows, noting that our anomaly is located below station height :

Mass of differential element, $dm = (r\, d\varphi)\, dr dz$
so that

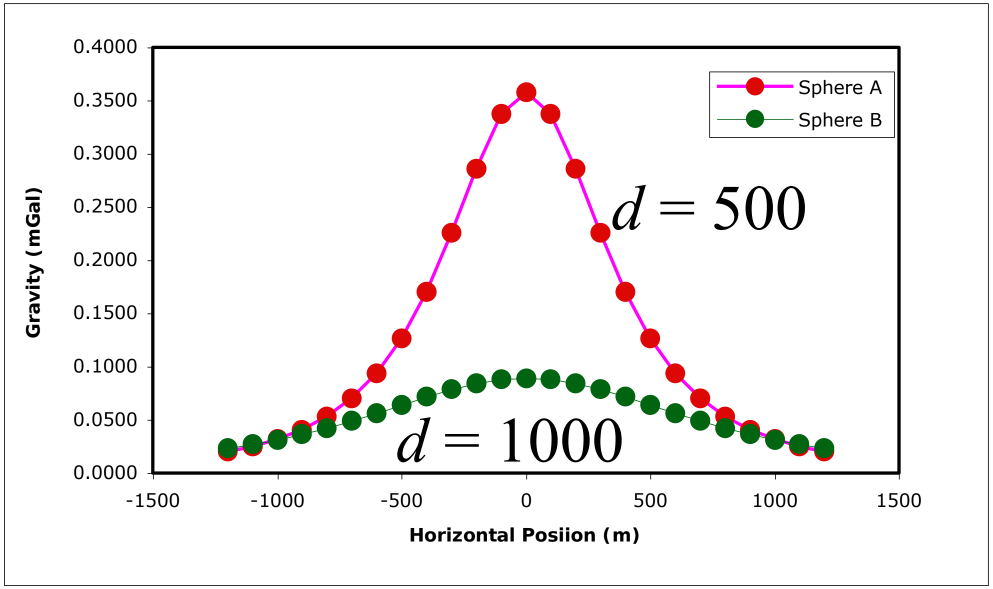
$$dg_{cyl} = -\frac{Gdm}{(r^2+z^2)}\cos\theta = -\frac{G(\rho\, r\, drdzd\varphi)}{(r^2+z^2)}\left(\frac{z}{\sqrt{(r^2+z^2)}}\right) = -G\rho\left(\frac{(rdr)(zdz)d\varphi}{\sqrt{(r^2+z^2)^3}}\right) \qquad (2a)$$

So,

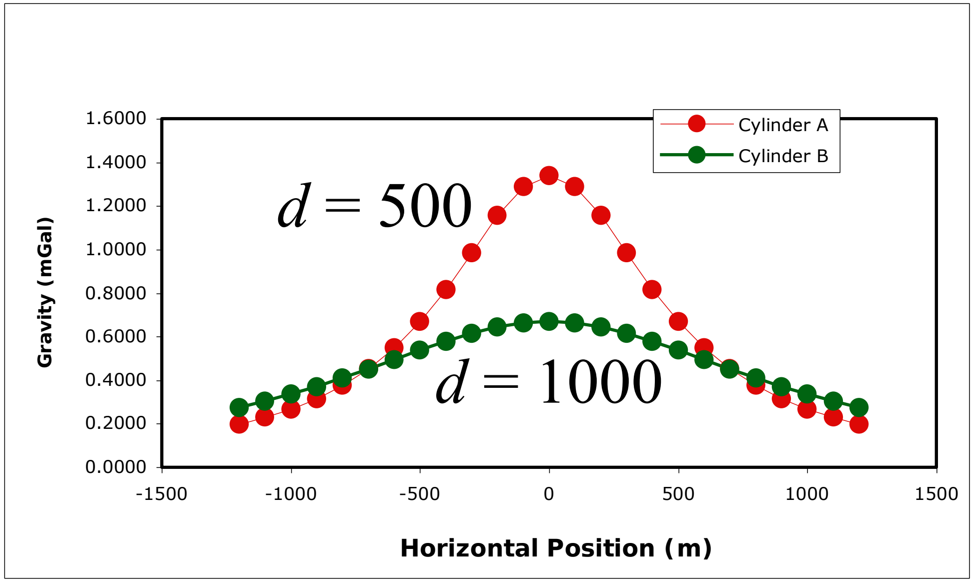
$$\begin{aligned}
\delta g_{cyl} &= -G\rho\int_{\varphi=0}^{2\pi} d\varphi \,.\int_{r=0}^{R}\left(\int_{z=h_1}^{h_2}\frac{zdz}{\sqrt{(r^2+z^2)^3}}\right) rdr \\
&= -2\pi G\rho\int_{r=0}^{R}\left(\frac{r}{\sqrt{(r^2+h_1^2)}} - \frac{r}{\sqrt{(r^2+h_2^2)}}\right)dr \\
&= -2\pi G\rho\left[\left(\sqrt{(0+h_1^2)} - \sqrt{(0+h_2^2)}\right) - \left(\sqrt{(R^2+h_1^2)} - \sqrt{(R^2+h_2^2)}\right)\right] \\
&= 2\pi G\rho\left[\left(\sqrt{(R^2+h_1^2)} - h_1\right) - \left(\sqrt{(R^2+h_2^2)} - h_2\right)\right] \qquad (2b)
\end{aligned}$$

In principle can calculate an anomaly for a density anomaly with any arbitrary body shape using:

$$\delta g_z = \iiint_V \left\{ \frac{G\,\delta\rho}{r^2}\,\hat{r} \right\} \cdot \hat{k}\, dV$$



$\Delta\rho = 400,\ R = 200$



Recall for a sphere:

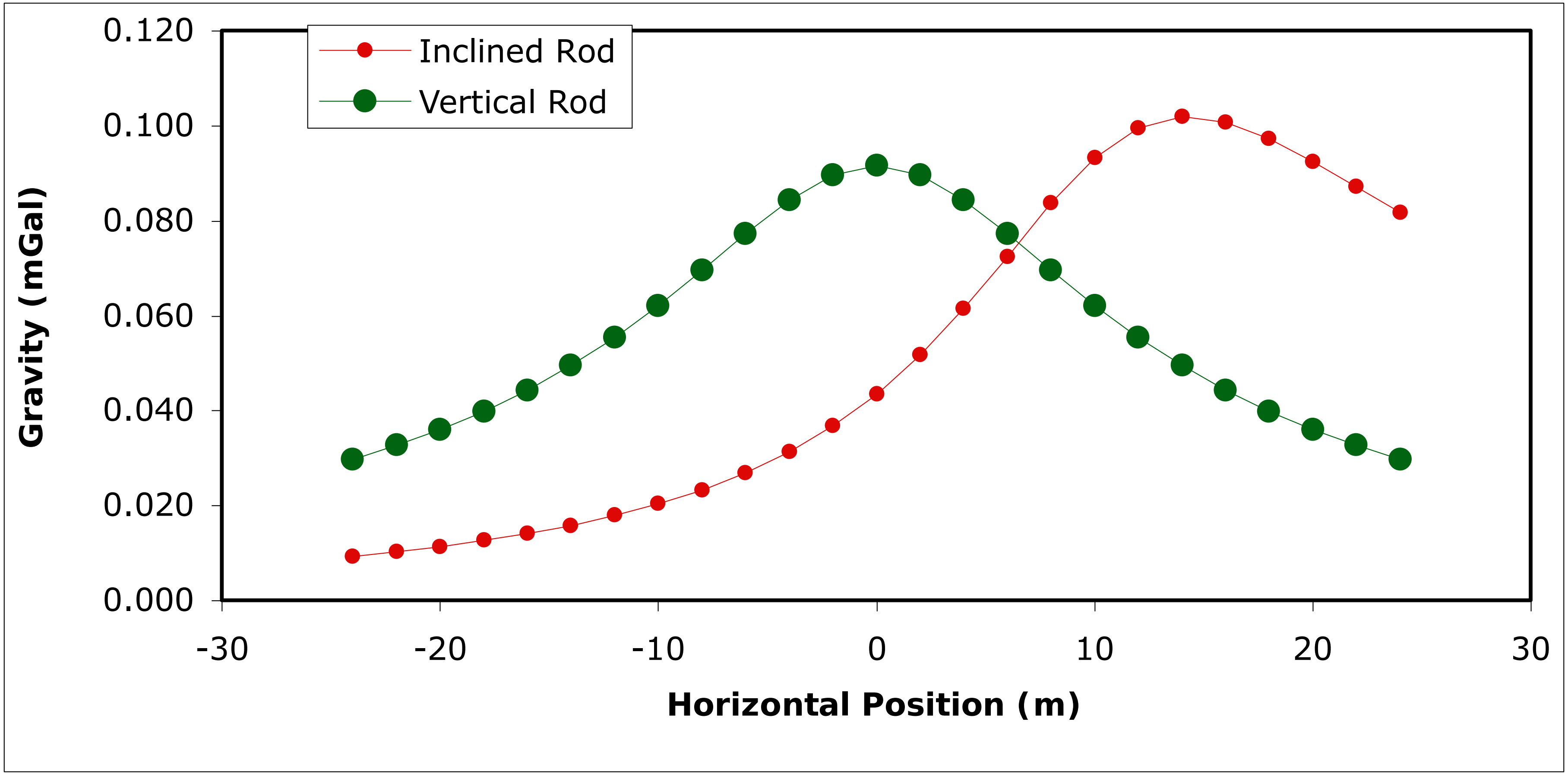
$$\Delta g_z = \frac{4}{3}\pi G R^3 \Delta\rho \frac{d}{\left(x^2 + d^2\right)^{\frac{3}{2}}}$$

∞ horizontal cylinder:

$$\Delta g_z = 2\pi G R^2 \Delta\rho \frac{d}{\left(x^2 + d^2\right)}$$

Vertical cylinder (top at h_1, bottom at h_2):

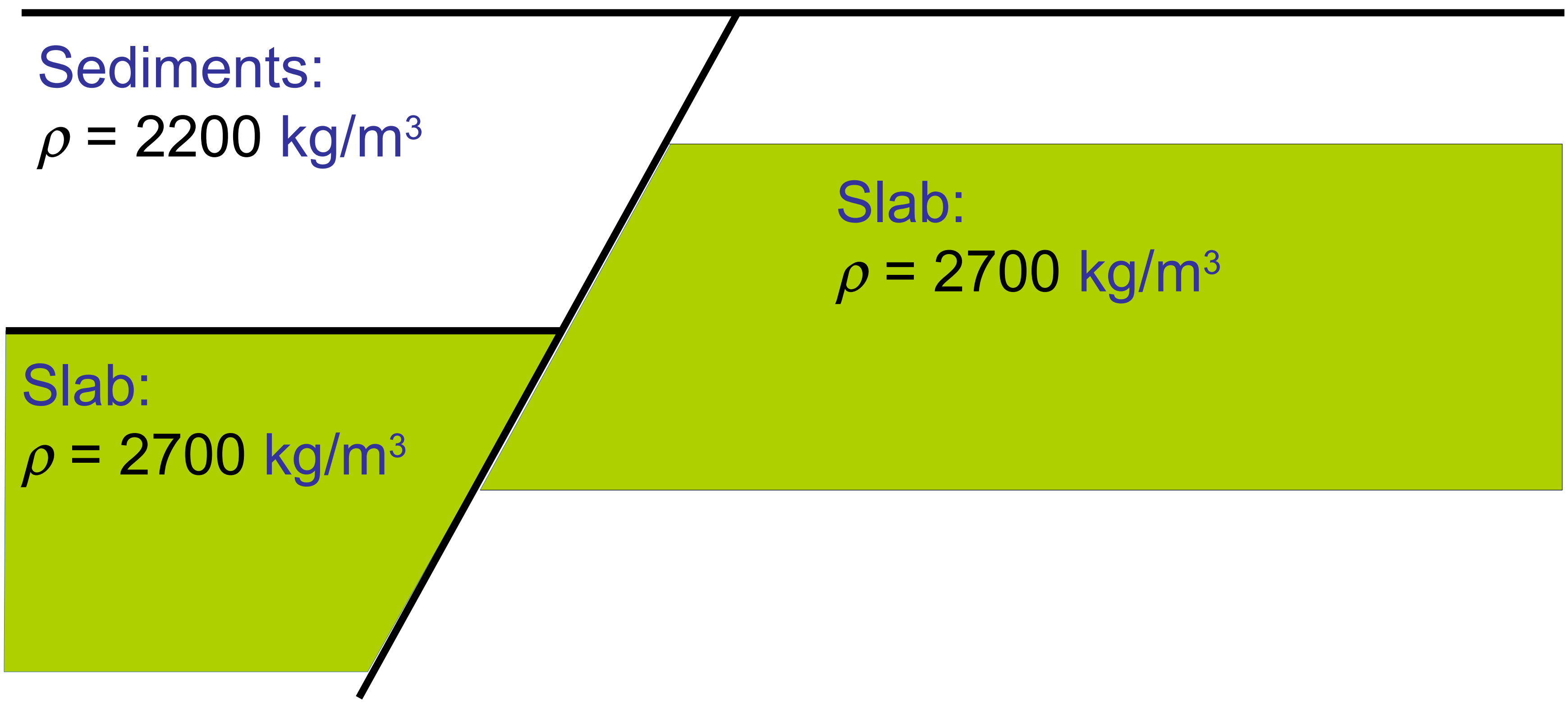
$$\Delta g_z = 2\pi G \Delta\rho \left(h_2 - h_1 + \sqrt{R^2 + h_1^2} - \sqrt{R^2 + h_2^2} \right)$$



Gravity Anomaly: Fault offset slab

Recall:

$$\delta g_z = \iiint_V \left\{ \frac{G\delta\rho}{r^2} \hat{r} \right\} \cdot \hat{k}\, dV$$



Gravity Anomaly: Fault offset slab

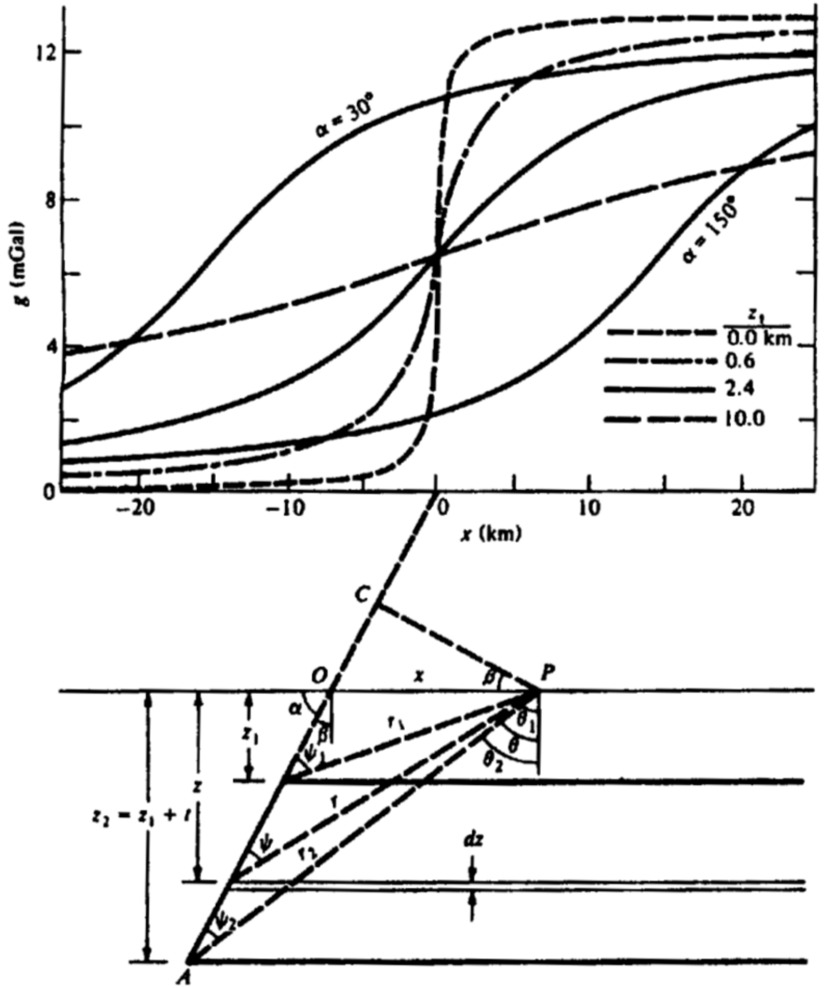


Figure 2.30. Gravity effect of a semiinfinite slab. t = 300 m, α = 90° except where otherwise noted on the curves, ρ = 1 gm/cm³.

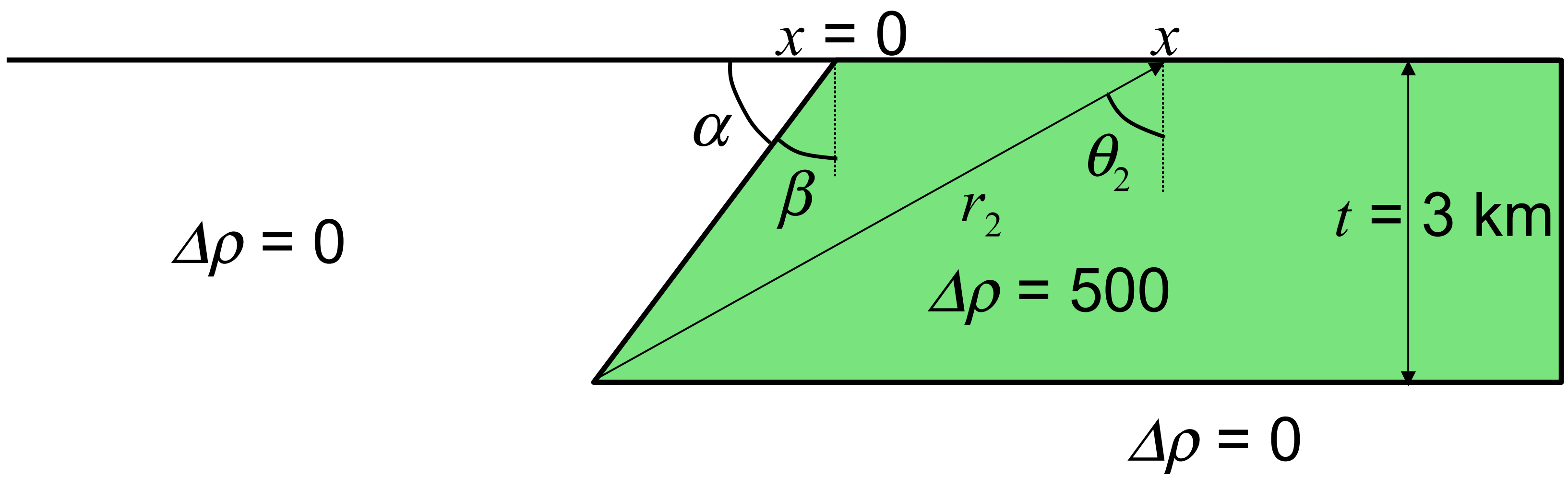
Substituting in Equation (2.68) and noting that $t = (z_2 - z_1)$, we obtain

$$g = 2\gamma\rho\{(\pi/2 + \beta)t + (\theta_2 - \beta) \times (z_2 + x\sin\beta\cos\beta) - (\theta_1 - \beta)(z_1 + x\sin\beta\cos\beta) + x\cos^2\beta\ln(r_2/r_1)\}$$

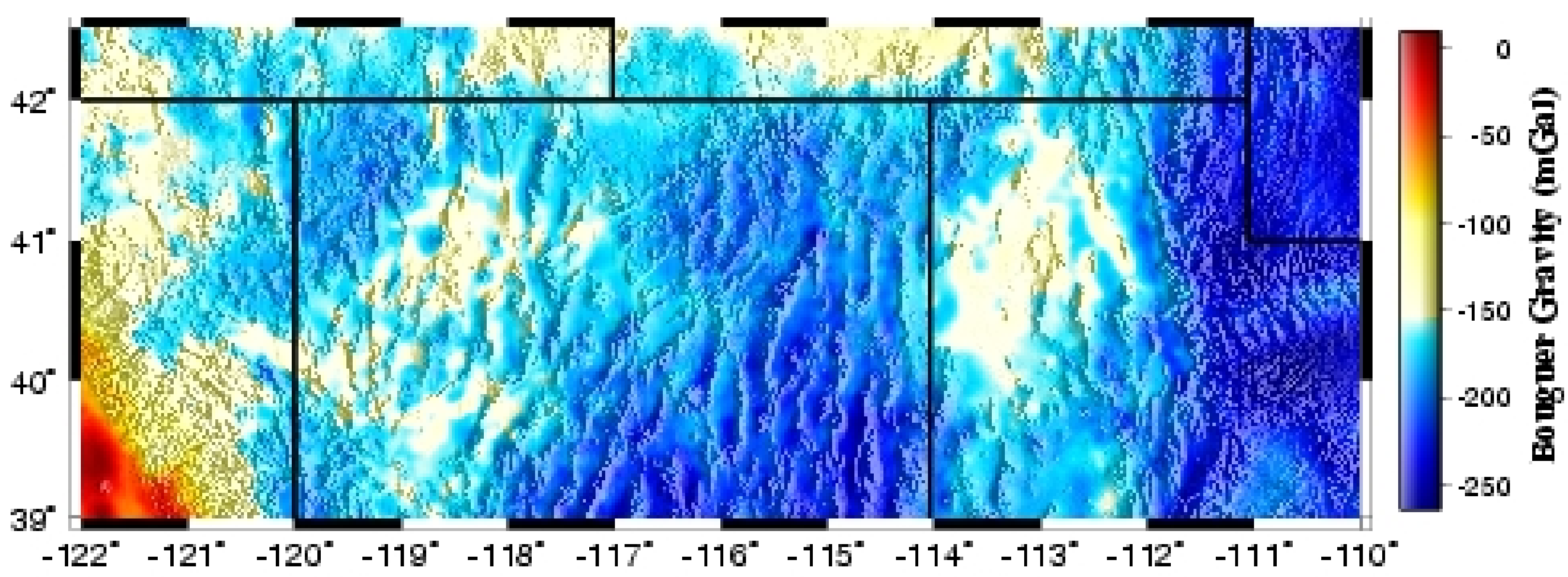
$$\Delta g_{fault} = 2\gamma\rho\{(\pi t/2) + (z_2\theta_2 - z_1\theta_1) + x(\theta_2 - \theta_1)\sin\beta\cos\beta + x\cos^2\beta\ln(r_2/r_1)\} \quad (2.69)$$

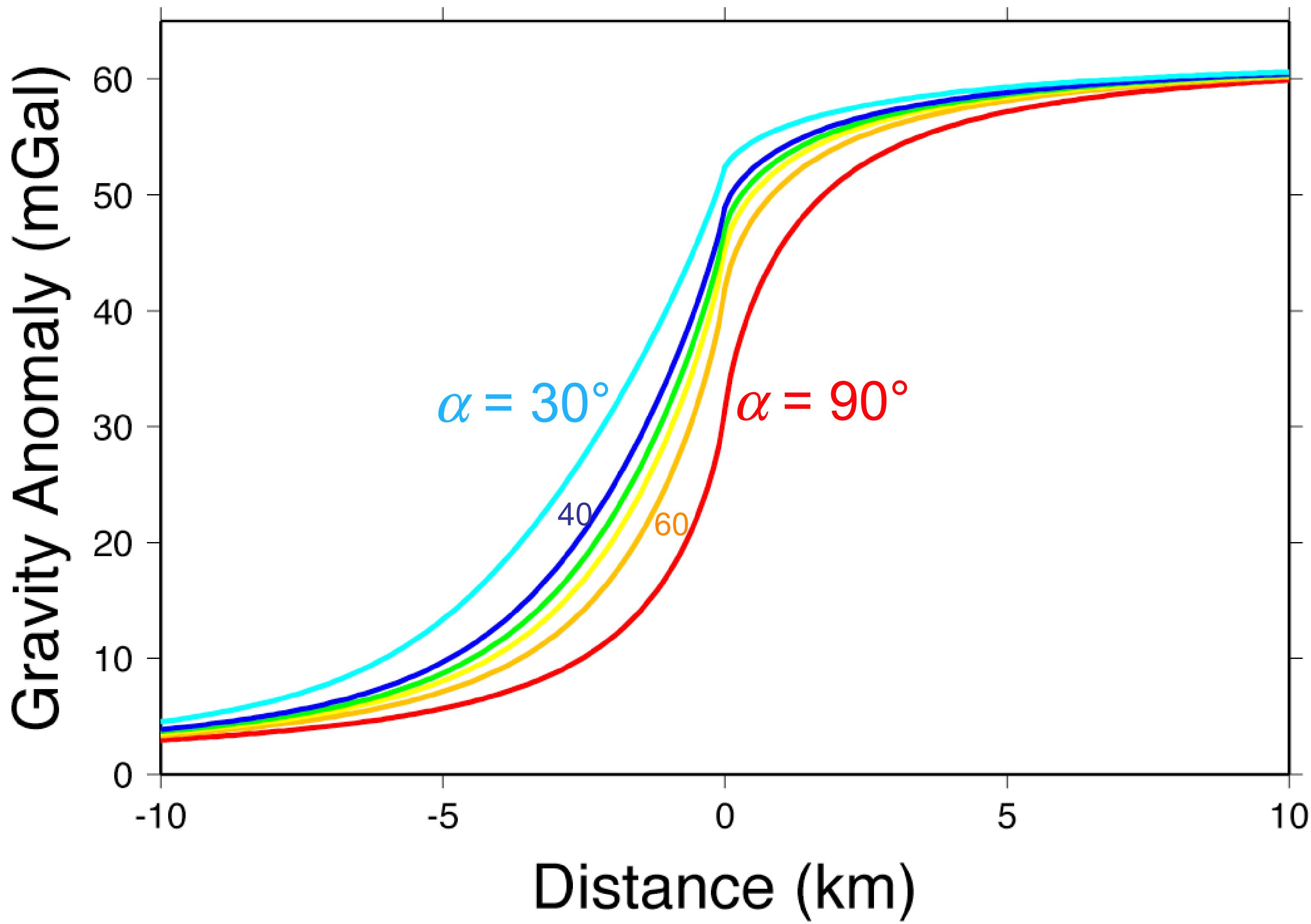
Applied Geophysics, 2nd ed, Telford et al. 1990

Gravity Anomaly: Surface Fault Offset Layer

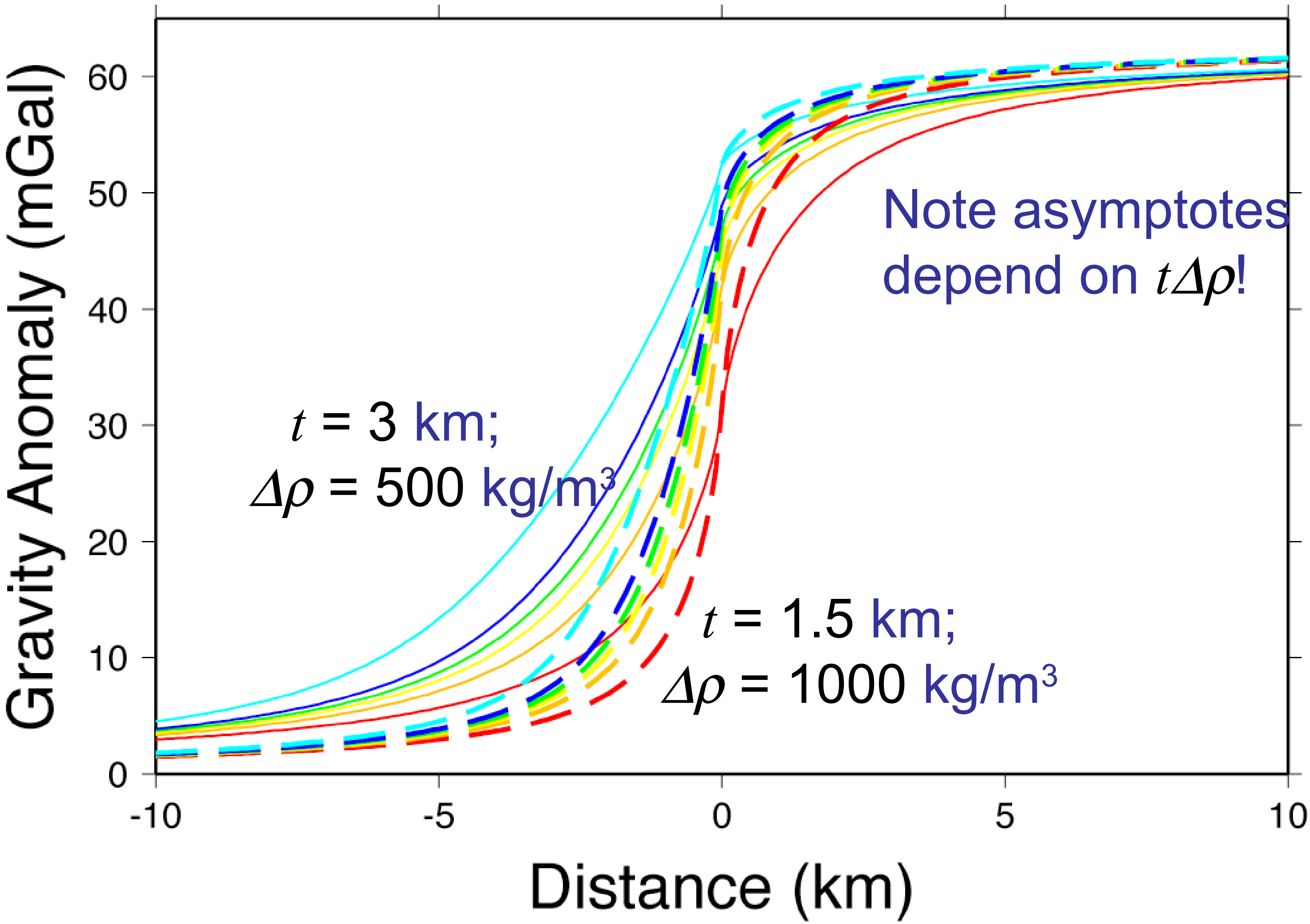


$$g = 2G\Delta\rho\left[\frac{\pi t}{2} + \theta_2 t + x\left(\theta_2 - \frac{\pi}{2}\right)\sin\beta\cos\beta + x\cos^2\beta\ln\left(\frac{r_2}{x}\right)\right]$$





Gravity Anomaly (mGal)
60
50
40
30
20
10
0
-10
-5
0
5
10
Distance (km)
α = 30°
α = 90°
40
60



Gravity Anomaly (mGal)
60
50
40
30
20
10
0
-10
-5
0
5
10
Distance (km)
Note asymptotes depend on tΔρ!
t = 3 km; Δρ = 500 kg/m3
t = 1.5 km; Δρ = 1000 kg/m3