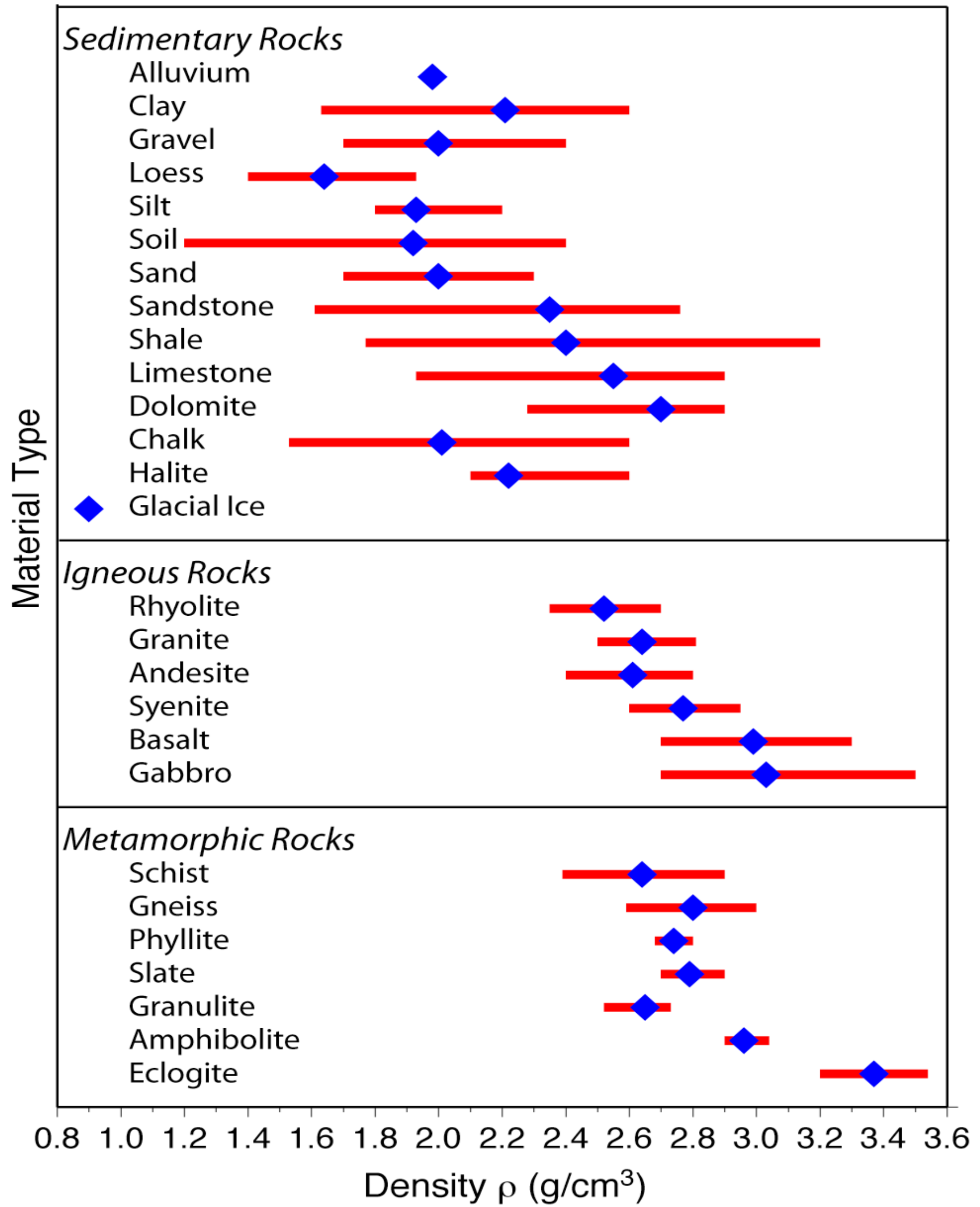
Gravity Modeling

Seeks to characterize subsurface structures via geometry of variations in mass density ρ…

ρ depends on rock type, porosity and pore fluids, temperature

Generally ρ lowest for soils/seds, higher for lithified sed rx, highest for crystalline rx, increases with depth



Estimating Gravity Anomalies

Most structural studies use terrestrial (or surface-based) gravimeter measurements, which will include the effects from the following:

- Distance to Earth's center of mass: g_{normal}, δg_{tides}, $\delta g_{Free\text{-}Air}$
- KNOWN mass attractions: atm., δg_{atm}, water-table, δg_{H_2O}, topography, $\delta g_{Terrain}$ & $\delta g_{Bouguer}$
- UNKNOWN mass attractions of interior density: Δg_i

We are only interested in the last contribution. So, we subtract out the KNOWN contributions from the measured gravity, g_m:

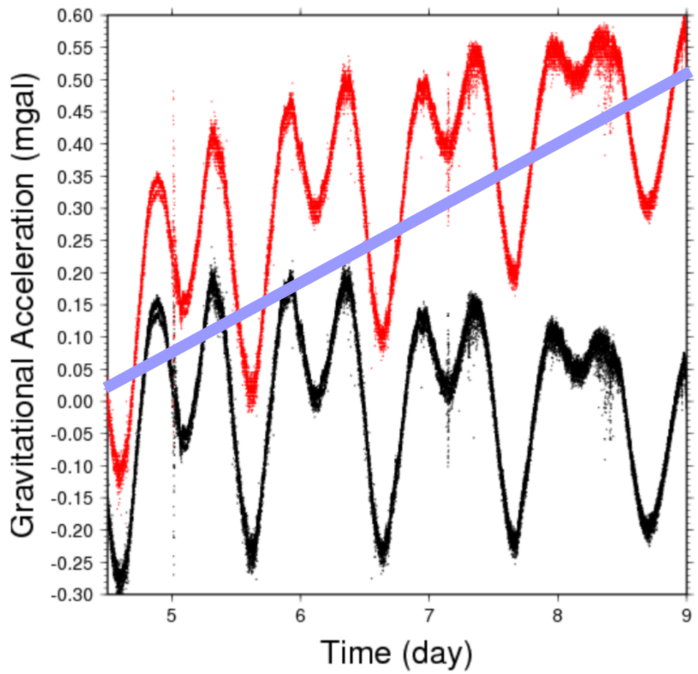
$$\Delta g_i = g_m - g_{normal} + \delta g_{tides} + \delta g_{Eotvos} - |\delta g_{atm,h}| + \delta g_{atm,\Delta p} + \delta g_{Free\text{-}Air} - \delta g_{Bouguer} + [\, |\Sigma_i\, \delta g_{Terrain,i}| + \delta g_{H_2O} \,]$$

$h_{tide} > h_0$ $v_{east} > 0$ $\Delta p > 0$ $h > h_0$ $h > h_0$ $h_{H_2O} > h_0$

NOTE: signs for δg assume:

- **Positive height** of measuring station, h, above ellipsoid height, h_0
- **Eastward velocity** at the Earth's surface w.r.t. its inertial (moving) reference frame, v_{east} is positive

Instrument Drift (Instrument specific)



A continuous record of gravitational acceleration in the USF lab, gathered at 1 s intervals for several days. The actual drift curve is shown in red, the detrended drift curve s shown in black.The drift curve (red) can be thought of as consisting of two components. The Earth tide varies predominantly on approximately 12 and 24 hr cycles. The tide develops longer period variation (a beat) due to the elliptical motion of the Sun and Moon.

(Courtesy: Chuck Conner - USF)

Instrument drift is the variation in gravity ***only*** due to the fact that the gravimeter registers different readings with time, due to:

- mechanical,
- thermal, and
- electrical

changes within the instrument

Tidal beats are due to resonance between diurnal solar and semi-diurnal lunar tides. The beat period is just over 14 days, or twice a month!

Solid Earth Tides (~ 0.3 mGal / 300 μGal)

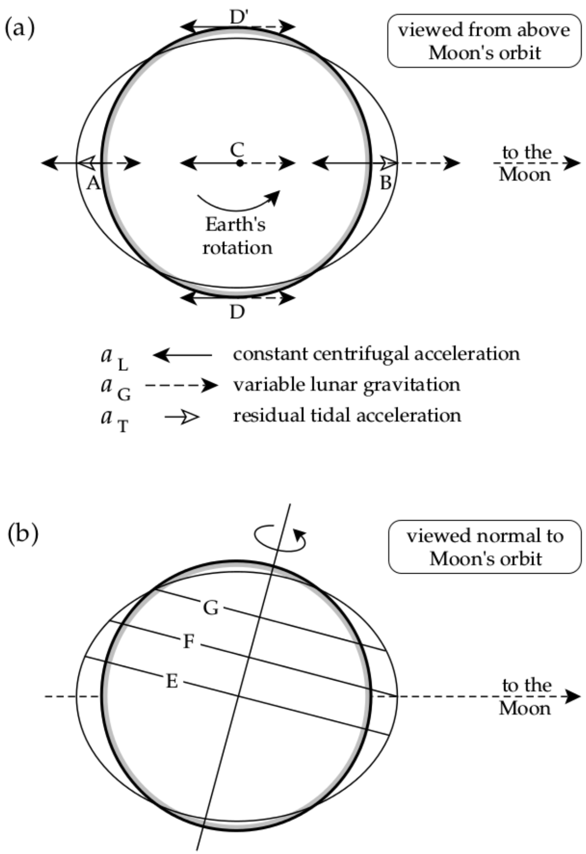


Fig. 2.11 (a) The relationships of the centrifugal, gravitational and residual tidal accelerations at selected points in the Earth. (b) Latitude effect that causes diurnal inequality of the tidal height.

W. Lowrie, 2000

- Solid Earth shape changes due to tidal attractions from nearby heavenly bodies (dominated by the Moon & Sun).
- Peak tidal acceleration is the difference between the Moon (or Sun)'s pull at the centroid of the Earth and its surface (nearside/far-side):

Tidal Correction:

nearside ⇒ *towards Moon / Sun*; *horizon* ⇒ 90° from *Earth – Moon / Sun line*; *both along the Ecliptic plane*

Maximum Tidal Correction (nearside):

$$\delta g_{tidal,nearside} = \frac{GM_i}{r_{ei}^2}\left\{\left(1-\frac{R_E}{r_{ei}}\right)^{-2} - 1\right\} \approx \frac{GM_i}{r_{ei}^2}\left\{2\left(\frac{R_E}{r_{ei}}\right) + 3\left(\frac{R_E}{r_{ei}}\right)^2 + \ldots\right\}$$

Minimum Tidal Correction (horizon):

$$\delta g_{tidal,horizon} = -\frac{GM_i}{r_{ei}^2}\sin\delta \ \ (towards\ Earth - centroid) \approx -\frac{GM_i}{r_{ei}^2}\left(\frac{R_E}{r_{ei}}\right)$$

where, M_i *is the mass of the Moon* $(0.0123\, M_E)$ or *Sun* $(3.333\times10^5\, M_E)$: $M_E = 5.97\times10^{24}\, kg$;
$G = 6.674\times10^{(-11)}\, m^3\cdot kg^{-1}\cdot s^{-2}$; R_E *the mean Earth radius* $(6.371\times10^6\, m)$, &
r_{ei} *the centroidal distance* from *Earth – Moon* $(3.82\times10^8\, m)$ or *Earth – Sun* $(1.496\times10^{11}\, m)$.

MOON: $\delta g_{t,n} \approx 0.115\, mGal$ $\quad \delta g_{t,h} \approx -0.057\, mGal$; &

SUN: $\delta g_{t,n} \approx 0.051\, mGal$ $\quad \delta g_{t,h} \approx -0.026\, mGal$;

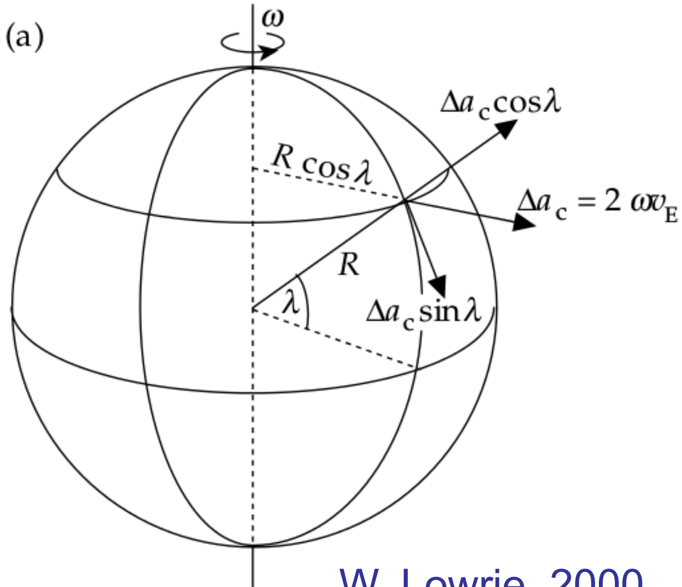
So, $\sum_i \left|\delta g_{t,i}\right| \approx 0.3\, mGal$ *total*.

- Very well understood. So, for these corrections, record: lon-lat, date, and time of measurement
- Can use well established solid earth tidal models

  (e.g., CSR 3.0 - Eanes & Bettadur, 1995; ETERNA 3.30 – Wenzel et al 1996; Matthews et al., 1997).
- `https://geodesyworld.github.io/SOFTS/solid.htm`
- `https://github.com/hydrogeoscience/pygtide`

Eotvos Correction

(Varies, ~ 10s of mGal to 1000s of mGal)



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Due to the East-/West-ward motion of gravity station in an inertial frame (e.g., ship, aircraft):

$$\ddot{E}\ddot{o}tv\ddot{o}s\ Correction: For\ moving\ measurement\ station\ (e.g., ship\ \mathrm{or}\ aircraft)$$

$$\delta g_{Eotvos} = 2\,\omega\, v_{east} \cos\lambda = \frac{4\pi v_{east}}{T} \cos\lambda$$

$$where, v_{east}\ is\ positive\ \&\ v_{west}\ is\ negative.\quad Typical\ corrections\ at\ 45^{o}\ latitude:$$

$$\approx\ 29\,mGal\quad for\ a\ ship\ sailing\ at\ v_{east} = 10\,km/h\quad !$$

$$\approx\ 856\,mGal\quad for\ a\ plane\ flying\ at\ v_{east} = 300\,km/h\quad !!$$

Atmospheric & Hydrologic Corrections
(~ 50 μGal EACH)

- Mass of atmosphere above (primarily a function of Lapse rate, and height) We assume a "Bouguer" slab (later slide) of air of height, *h*. *Then,*

Ideal gas approximation with linear tropospheric lapse rate $(<11\,km\ height)$:

$$\delta g_{atm} = -2\pi G h \rho_a = -2\pi G h \rho_0 \left[\frac{T_0}{T_0 + Lh}\right]^{\left(1+\frac{g_0 M}{RL}\right)} \quad decreases \text{ for } positive\ elevation\ change, h$$

$$where, \qquad g_0 = 9.8067\left[\frac{m}{s^2}\right]; \; T_0 = 288.15\left[{}^0K\right]; \; \rho_0 = 1.225\left[\frac{kg}{m^3}\right];$$

$$L = -0.0065\left[\frac{{}^0K}{m}\right]; \; R = 8.3145\left[\frac{Nm}{mol - {}^0K}\right]; \; M = 0.02896\left[\frac{kg}{mol}\right]$$

$$\approx -0.05137\,h\left[\frac{288.15}{288.15 - 0.0065\,h}\right]^{-4.255787} \mu Gal, h[m]$$

$$\approx -46.62 \; \mu Gal \;\; at\ h = 1\,km;$$

$$\approx -84.41 \; \mu Gal \;\; at\ h = 2\,km; \quad \approx -114.37 \; \mu Gal \;\; at\ h = 3\,km$$

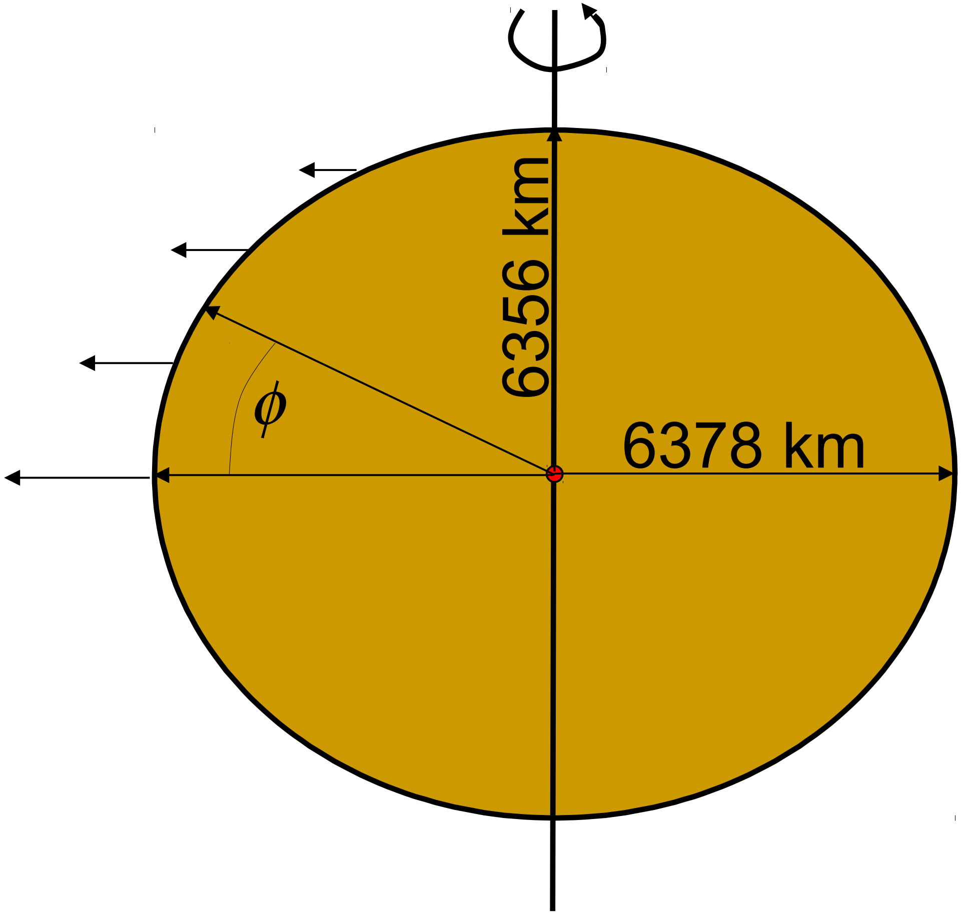
$$\approx -167.87 \; \mu Gal \;\; \text{for } the\ entire\ Troposphere\,(h = 11\,km);$$

- Another pressure effect is associated with time-dependence of air mass above station: $\delta g_{atm(t)}$ ~ -3.0 Δp *nano-Gal* (record pressure in *Pa*).
  Higher pressure increases upward acceleration – for typical high & low pressure system changes of 5-10 kPa, this effect contributes to ***< 30 μ Gal***.

- Water-table below (function of rainfall, porosity, runoff, …) ~ 10-50 μGal

Normal Gravity g_n (Reference Ellipsoid)

(~ 5186 mGal {!!} higher at poles than equator)



Distance r from Earth's center of mass is a function of latitude, so we subtract from the total, gravity due to ***ellipsoidal figure of the earth***.

Earth's rotation induces an outward (centrifugal – non-inertial) acceleration as a function of radial distance from axis of rotation, so we correct for this also.

Excess mass due to the equatorial bulge adds to the gravity at the Equator.

These three effects result in **~ 5.186 Gal** higher gravity at the poles compared to that at the equator

Normal Gravity g_n (Reference Ellipsoid)

Geopotential:

$$\psi_g = \overbrace{-\frac{GM_E}{r}\left\{1 - \underbrace{\left(\frac{I_p - I_e}{M_E R_E^2}\right)^2}_{J_2}\left(\frac{R_E}{r}\right)^2 p_2(\cos\theta) - J_3\, p_3(\cos\theta)\left(\frac{R_E}{r}\right)^3 - \ldots\right\}}^{Gravitational} + \overbrace{\left(\frac{1}{2}\right)\omega^2 r^2 \sin^2\theta}^{Centrifugal}$$

where M_E *is the mass of the Earth* $(5.97\times 10^{24}\,kg)$; R_E *the mean Earth radius* $(6.371\times 10^{6}\,m)$;

I_p, I_e *are moments of inertia of the Earth about its polar (i.e., rotational), and equatorial axes.;* &

$p_n(\cos\theta)$ *are Legendre Polynomials:* $p_2(\cos\theta)=\frac{1}{2}\left(3\cos^2\theta-1\right)$; $p_3(\cos\theta)=\frac{1}{2}\cos\theta\left(5\cos^2\theta-3\right)$; ...

Normal Gravity:

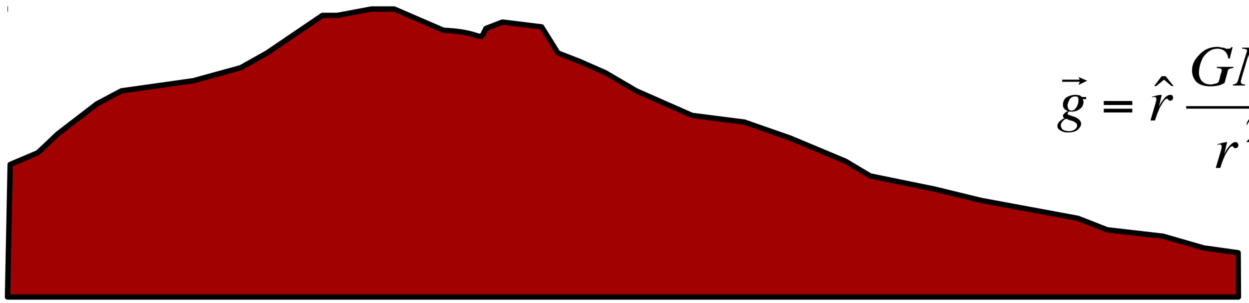
$$g_n = \nabla\psi_g\cdot\hat{n} \approx g_0\left(1 + \beta_1\sin^2\lambda + \beta_2\sin^2 2\lambda\right)$$

where $g_0 = g_e = 9.780318\,m/s^2$ $(equator, \lambda=0^o)$; $\beta_1 = 5.3024\times 10^{-3}$; $\beta_2 = -5.87\times 10^{-6}$;

so, $g_p = 9.832177\,m/s^2$ $(poles, \lambda=\pm 90^o)$ $\Rightarrow$ $\delta g_{n,p} \approx 5186\,mGal$!!!

(1 μGal accuracy ⇒ 1.25 m latitude accuracy!)

Free Air Correction (~ 0.3086 mGal/m)



$$\vec{g} = \hat{r}\,\frac{GM}{r^2}$$

Following topography during a survey changes the radial distance to the Earth's center of mass. Correction for this change is elevation is called ***"free air":***

For a spherical Earth approximation,

$$\delta g_{free-air} = \frac{dg}{dr}\delta r = -2\frac{GM}{r^3}h = -2\frac{GM}{r^2}\frac{h}{r} = -\frac{2gh}{r}, \quad \textit{so decreases for positive } h$$

$$\textit{Setting } r = R_E: \quad min: 6356\,km;\ mean: 6371\,km;\ max: 6378\,km,$$

$$\textit{where } g\textit{-values are}: max: 9.83\,m/s^2;\ mean: 9.81\,m/s^2;\ min: 9.789\,m/s^2;$$

$$\delta g_{free-air} \approx 0.3067\,h \;\; mGal \;\; (equator) \text{ to } 0.3093\,h \;\; mGal \;\; (poles);$$

$$\textit{Positive for } h > h_{ellipsoid} \Rightarrow \textit{added to measured gravity}.$$

For the more accurate Ellipsoidal Earth approximation,

$$\delta g_{free-air} = \frac{dg}{dr}\delta r = \left[0.3086 + 2.3\times10^{-4}\cos 2\lambda - 2\times10^{-8}h\right] h \;\; mGal$$

$$(h \text{ is in } m;\ \lambda \textit{ is latitude})$$

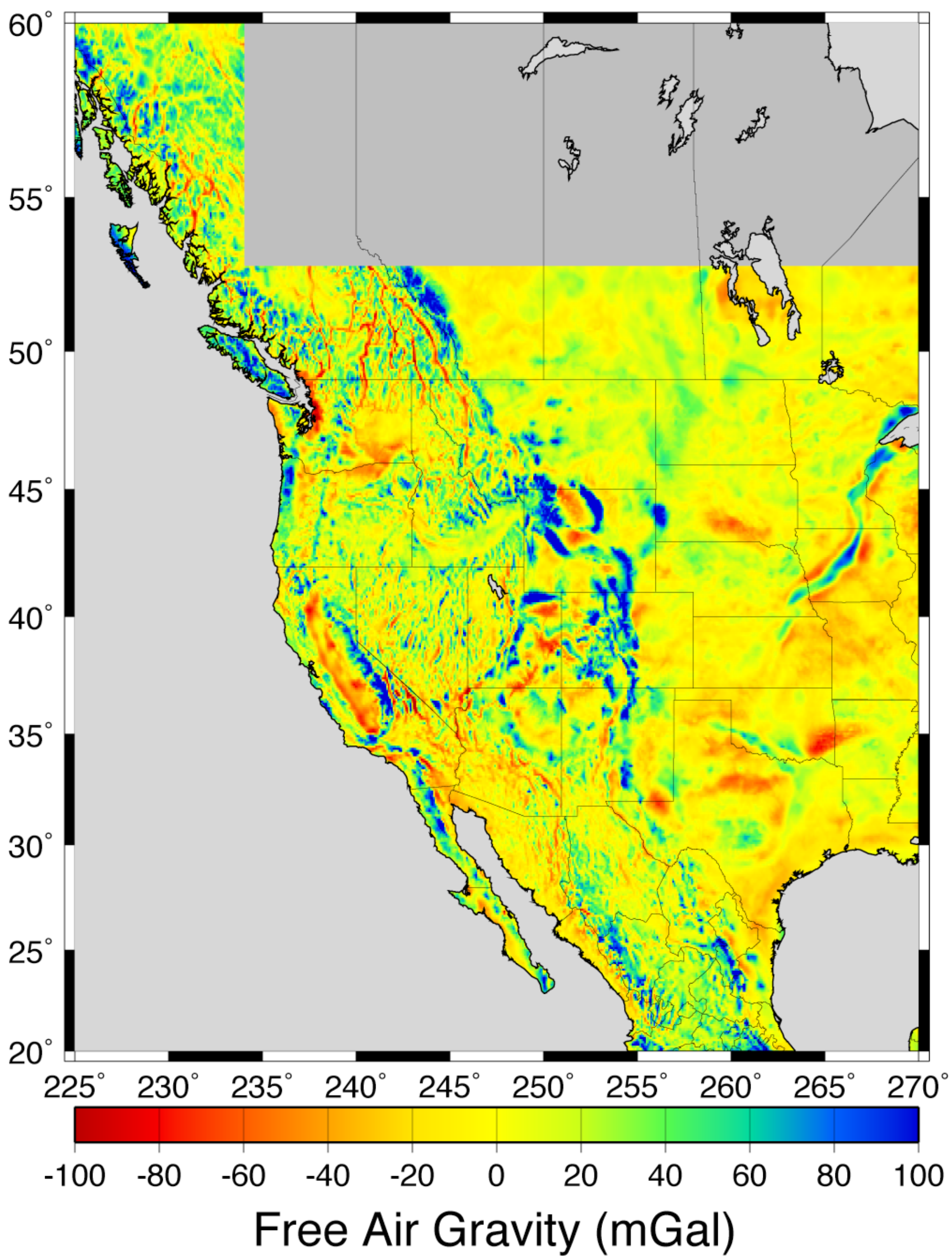
(1 μGal accuracy ⇒ 3 mm height accuracy!)

Free Air Anomaly

After observed gravity measurement is corrected for free air and other effects talked about so far, we call it the ***free air anomaly:***

$$\Delta g_{FA} = g_m - g_{normal} + \delta g_{tides} + \delta g_{Eotvos} - |\delta g_{atm,h}| + \delta g_{atm,\Delta p} + \delta g_{Free\text{-}Air}$$

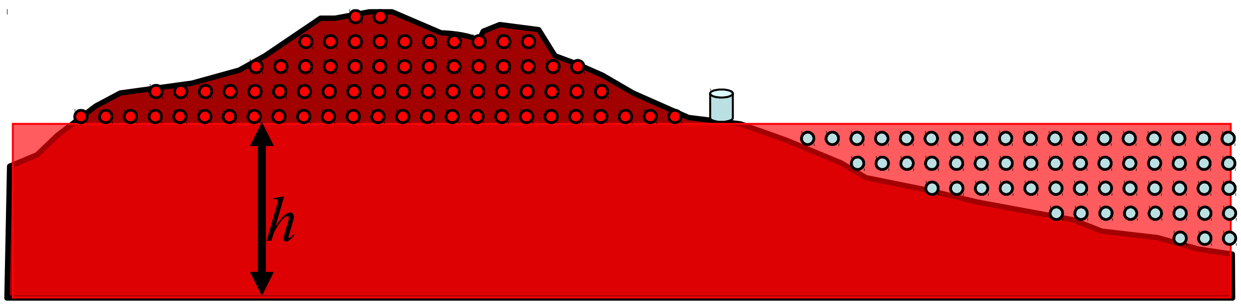
At right, millions of terrestrial free air Relative measurements have been referenced to a small ***network*** of Absolute gravity stations and ***adjusted*** to a common reference.



(courtesy, Tony Lowry)

Topographic Mass: Bouguer Correction

(~ -0.112 mGal/m)



Simple Bouguer: Approximate topography as a slab with thickness h:

Bouguer Correction: Terrain correction with z increasing downward; $\varphi_0 = 2\pi$; $r_1 = 0$; & $r_2 \rightarrow \infty$
With these parameter substitutions,

$$\delta g_{bouguer} = -2\pi G\rho\left[\left(\sqrt{(h^2)}\right) - (0)\right] = -2\pi G\rho h, \text{ negative for positive elevation change}$$

For elevation above the ellipsoid, the "excess" mass below the station adds to the ellipsoidal gravity. So, the Bouguer correction for ***positive elevation change, h, is negative***. For standard ρ = 2670 kg/m^3, the simple Bouguer correction is ***-0.11195 mGal/m*** :

$$\Delta g_B = g_m - g_{normal} + \delta g_{tides} + \delta g_{Eotvos} - |\delta g_{atm,h}| + \delta g_{atm,\Delta p} + \delta g_{Free\text{-}Air} - \delta g_{Bouguer} + [\,|\Sigma_i\, \delta g_{Terrain,i}| + \delta g_{H_2O}\,]$$

Complete Bouguer: Additional correction for upward attraction of mass above; and "negative attraction" of missing mass below (dotted material & grey terms above) …

Topographic Mass: Terrain Correction

(varies)

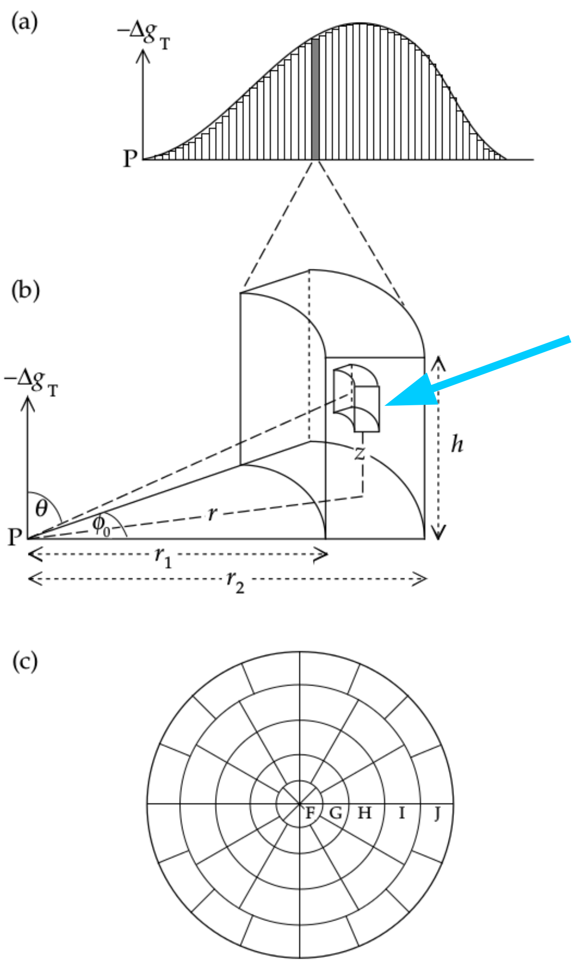


Fig. 2.38 Terrain corrections Δg_T are made by (a) dividing the topography into vertical elements, (b) computing the correction for each cylindrical element according to its height above or below the measurement station, and (c) adding up the contributions for all elements around the station with the aid of a transparent overlay on a topographic map.

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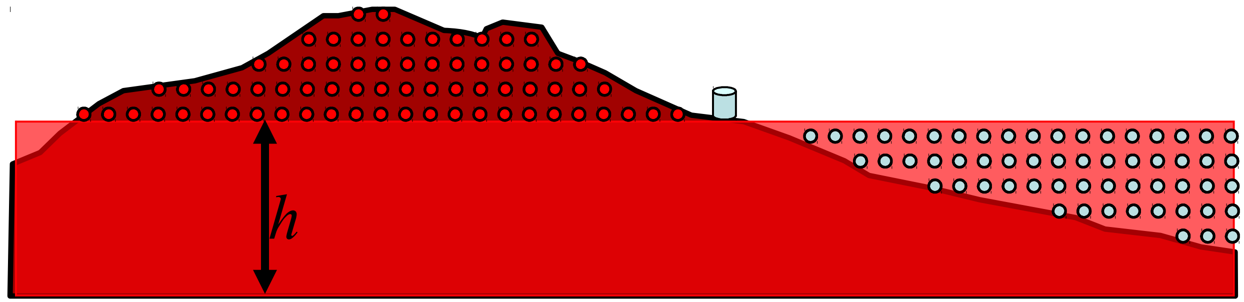


Terrain Correction (upward if $h>0$):
Mass of differential element, $dm = (r\,d\varphi)\,dr\,dz$
so that

$$dg_{terrain} = \frac{G\,dm}{(r^2+z^2)}\cos\theta = \frac{G(\rho\, r\, dr\,dz\,d\varphi)}{(r^2+z^2)}\left(\frac{z}{\sqrt{(r^2+z^2)}}\right) = G\rho\left(\frac{(r\,dr)(z\,dz)\,d\varphi}{\sqrt{(r^2+z^2)^3}}\right)$$

So,

$$\delta g_{terrain} = G\rho\int_{\varphi=0}^{\varphi_0} d\varphi \,.\int_{r=r_1}^{r_2}\left(\int_{z=0}^{h}\frac{z\,dz}{\sqrt{(r^2+z^2)^3}}\right) r\,dr$$

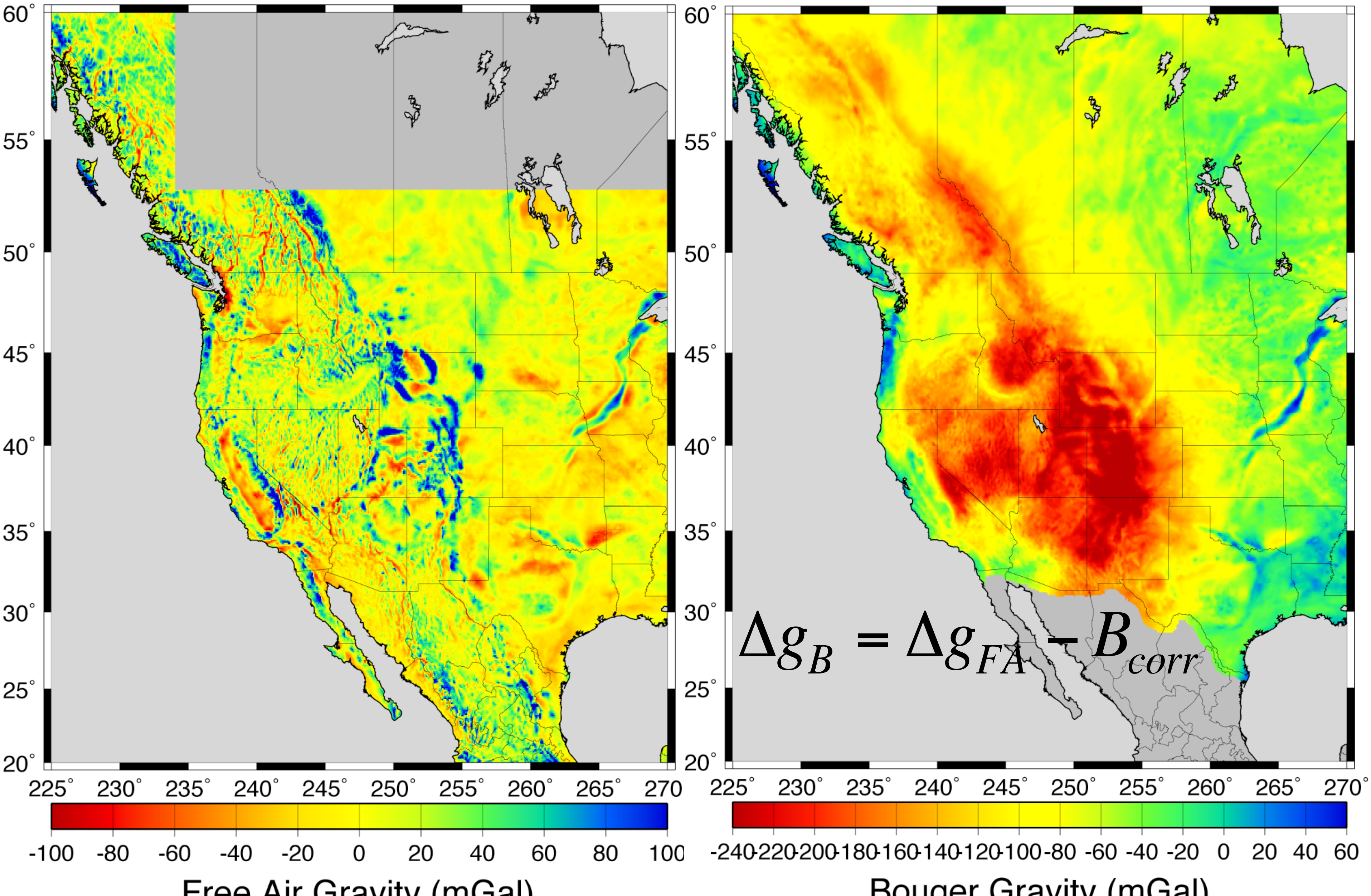
$$= G\rho\varphi_0\int_{r=r_1}^{r_2}\left(\frac{r}{\sqrt{(r^2+h^2)}} - 1\right)dr$$

$$= G\rho\varphi_0\left[\left(\sqrt{(r_1^2+h^2)} - r_1\right) - \left(\sqrt{(r_2^2+h^2)} - r_2\right)\right]$$

Bouguer Correction: Terrain correction with z increasing downward; $\varphi_0 = 2\pi$; $r_1 = 0$; & $r_2 \rightarrow \infty$
With these parameter substitutions,

$$\delta g_{bouguer} = -2\pi G\rho\left[\left(\sqrt{(h^2)}\right) - (0)\right] = -2\pi G\rho h,$$ *negative for positive elevation change*

Western North America: Bouguer Anomaly



(courtesy, Tony Lowry)

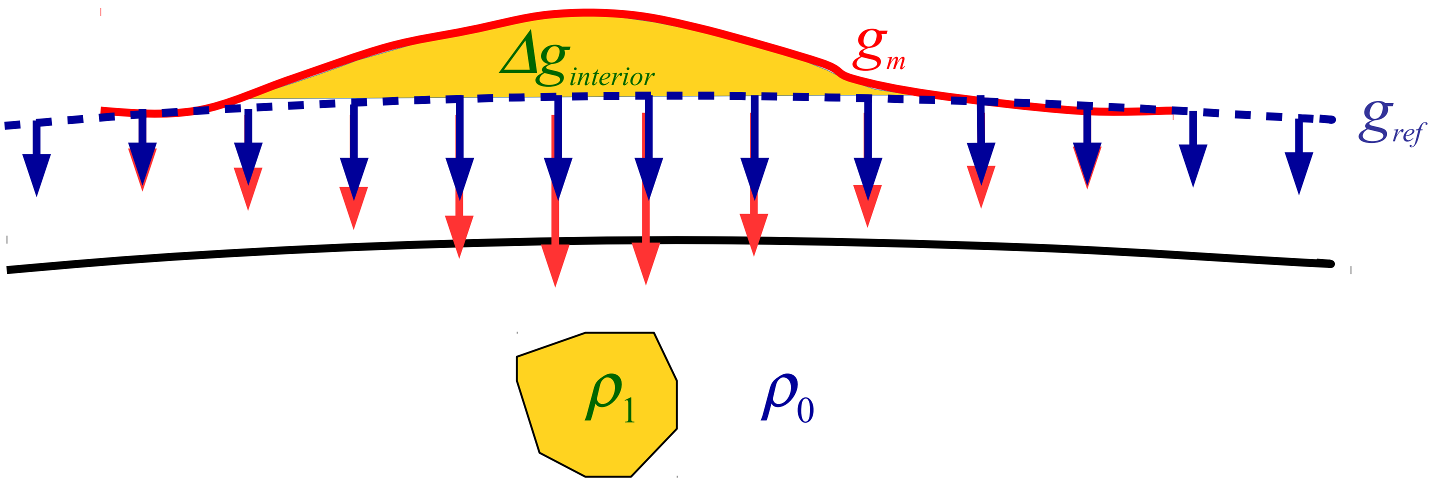
Interior Gravity Anomalies

We are now ready to estimate geometric shapes of interior density variations!

$$\Delta g_{interior} = g_m - g_{ref}$$

where,

$$g_{ref} = g_{normal} - \delta g_{tides} - \delta g_{Eotvos} + |\delta g_{atm,h}| - \delta g_{atm,\Delta p} - \delta g_{Free\text{-}Air} + \delta g_{Bouguer} - [\, |\Sigma_i\, \delta g_{Terrain,i}| + \delta g_{H_2O} \,]$$



$$\rho_1 > \rho_0$$