

## Quick Intro to Tensor Notation:

For our purposes, it is perhaps best to think of a tensor as a special kind of matrix for describing the multi-dimensional state (stresses, strains, elastic properties, etc.) acting on some object. Tensor's have some other special mathematical properties, but they aren't relevant for now.

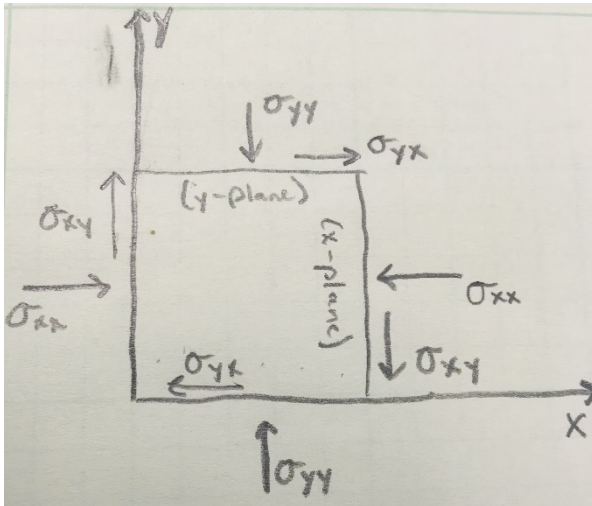
$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} \lambda\Theta + 2\mu e_{xx} & 2\mu e_{xy} \\ 2\mu e_{yx} & \lambda\Theta + 2\mu e_{yy} \end{bmatrix}$$

This is the stress tensor, which describes the stress acting in multiple dimensions (in this case, 2D) on an object. The tensor is comprised of **components** that follow **"on-in"** notation. For example,  $\sigma_{xx}$  describes the stress component acting **on** the x-plane (note the x-plane is the plane perpendicular to the x-axis), **in** the x-direction.  $\sigma_{xy}$  describes the stress acting on the y-plane, but the x-direction. Both  $\sigma_{xx}$  and  $\sigma_{yy}$  are oriented perpendicular to some plane (that's a **normal stress**, right?).  $\sigma_{xy}$  and  $\sigma_{yx}$  are acting parallel to some plane (which would be a **shear stress**, correct?).

The strain tensor, is as follows,

$$e = \begin{bmatrix} e_{xx} & e_{xy} \\ e_{yx} & e_{yy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2} \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \\ \frac{1}{2} \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) & \frac{\partial u_y}{\partial y} \end{bmatrix}$$

Similar to the stress tensor,  $e_{xx}$  and  $e_{yy}$  are normal strains (i.e., lengthening or shortening in a direction), and  $e_{yx}$  and  $e_{xy}$  are shear strains (think box becoming a parallelogram).



## Glossary of other variables:

$\lambda$	Lamé constant, related to other material moduli in a variety of ways
$\mu$	The second Lamé constant, also known as shear modulus. Describes response to an imposed shear stress.
$\nu$	Poisson's ratio, describes relationship of stress-parallel and -perpendicular strains. Think of a box compressing in one direction, and then lengthening in the other, it's the ratio of those two values
$\theta$	Dilation (HW p#2). Volume change in an object.
$\Theta_c$	Critical angle. Angle between incident ray and a line perpendicular to that surface
$\rho$	Density
$A$	Wave amplitude. Displacement of particle above or below a "reference" plane from the undisturbed object
$K$	Bulk modulus. Measure of resistance to compression under a <i>uniform</i> stress field (like underwater or even lithostatic).
$k$	Wave number. Wavelengths per unit distance ( $2\pi/\text{wavelength}$ ).
$\omega$	Angular frequency. Angular displacement per unit time ( $2\pi/\text{frequency in Hertz}$ ).
$u_x$	Displacement in the x-direction
$u_y$	Displacement in the y-direction
$V_p, V_s$	P- and S-wave velocity, respectively
$\hat{e}$	Average kinetic energy density ( $J/m^3$ ). The energy associated with particle motions as a wave passes through a material
$\hat{u}$	Average potential (strain) energy ( $J/m^3$ ). The energy stored within an object as it resists motion.
$\hat{e}$	Average total energy density ( $J/m^3$ )