Data Reduction (for surface measurements):

- Remove/avoid all metal objects when collecting data!!!
- Keep magnetometer high off the ground to reduce “noise”
- Sensitive to variations in ionosphere, magnetosphere: Perform looping and correct for drift; don’t bother measuring during solar storms!
- Correct for elevation if > a few hundred m (~0.03 nT/m)
- Horizontal position correction: *Use* ***WMM*** if latitude change is > a few hundred m (correction here ~6 nT/km)

From http://www.ngdc.noaa.gov/geomag-web/#igrfwmm

Component	Field Value	Secular Variation
Declination	11.724 degrees	-0.1072 °/year
Inclination	66.7027 degrees	-0.0219 °/year
Horizontal Intensity	20706.8 nT	-25.8 nT/year
North Component (x)	20274.8 nT	-17.4 nT/year
East Component (y)	4207.6 nT	-43.2 nT/year
Vertical Intensity (z)	48086.8 nT	-110.7 nT/year
Total Intensity	52355.6 nT	-111.9 nT/year

Magnetic Potential & Field in Magnetic Materials

Ampere – Maxwell Law for Magnetic Field:

$$\oint_C \vec{B}.\hat{t}\, ds = \mu I_{enclosed} \qquad (1a)$$

Applying Stokes' Theorem, and *expressing current* in *terms of area – density*:

$$\oiint_S (\vec{\nabla}\times\vec{B}).\hat{n}dA = \mu \oiint_S \vec{J}.\hat{n}dA$$

It can be shown that, $\vec{\nabla}\times\vec{B} = \mu\vec{J} = \mu(\overrightarrow{J_{free}} + \overrightarrow{J_{bound}}) = \mu(\overrightarrow{J_{free}} + \vec{\nabla}\times\vec{M}) \qquad (1b)$

where, *M is the magnetization per unit volume*, and *current density includes both free* and *bound contributions*.

$$so,\ \vec{\nabla}\times(\frac{\vec{B}}{\mu} - \vec{M}) = \vec{\nabla}\times\vec{H} = \overrightarrow{J_{free}} \qquad (1c)$$

where $\vec{H}$ *is the net magnetic field* in *the presence of magnetized media*. *If* $\overrightarrow{J_{free}} = 0$ (*High frequency approximation*),

then, $\vec{\nabla}\times\vec{H} = 0 \qquad (1d)$

So, there exists *a **scalar potential***, *W* (*no physical meaning as magnetic forces do NO work*):

$$\vec{H} = \vec{\nabla}W \qquad (1e)$$

$$Now,\ \vec{\nabla}\cdot\vec{H} = \vec{\nabla}\cdot(\frac{\vec{B}}{\mu} - \vec{M}) = \frac{1}{\mu}\vec{\nabla}\cdot\vec{B} - \vec{\nabla}\cdot\vec{M} \qquad (1f)$$

But from *Gauss' Law for magnetism*: $\oiint_S \vec{B}.\hat{n}dA = 0 = \iiint_V \nabla\cdot\vec{B}\,dV \Rightarrow \nabla\cdot\vec{B} = 0 \qquad (1g)$

so that: $\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M} \neq 0! \qquad (1h)$

and thus, from (1e) and (1h):

$$\nabla^2 W = -\vec{\nabla}\cdot\vec{M}, \qquad (POISSON's\ EQUATION!) \qquad (1i)$$

Magnetic Potential & Field in Magnetic Materials

$$\nabla^2 W = -\vec{\nabla}\cdot\vec{M}, \quad (POISSON's\ EQUATION!) \qquad (1i)$$

As with the scalar electrostatic and *gravity potentials, if we assume that magnetization is localized ($\rightarrow$ zero at ∞), the solution becomes (Helmholtz's Theorem):*

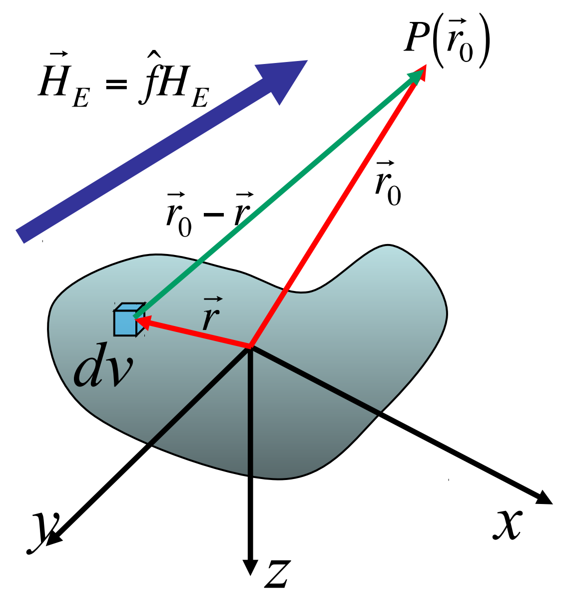
$$W(\vec{r}) = -\frac{1}{4\pi}\iiint_V \frac{\vec{\nabla}\cdot\vec{M}}{|\vec{r}-\vec{r}\,'|}dV' = \frac{1}{4\pi}\iiint_V \vec{M}\cdot\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)dV' \qquad (1j)$$

If $\vec{M}$ is a constant vector: $M_\alpha(n_x\hat{i}+n_y\hat{j}+n_z\hat{k})$, *then* $\vec{M}\cdot\vec{\nabla} = M_\alpha\frac{\partial}{\partial\alpha} = M_\alpha\left(n_x\frac{\partial}{\partial x}+n_y\frac{\partial}{\partial y}+n_z\frac{\partial}{\partial z}\right)$, and

$$W_\alpha(\vec{r}) = -\frac{M_\alpha}{4\pi}\frac{\partial}{\partial\alpha}\iiint_V \frac{dV'}{|\vec{r}-\vec{r}\,'|}, \quad \text{so that the net magnetic field,} \qquad (1k)$$

$$\Rightarrow \vec{H}_f(\vec{r}) = -\nabla_f W_\alpha = -\frac{M_\alpha}{4\pi}\frac{\partial^2}{\partial\alpha\,\partial f}\iiint_V \frac{dV'}{|\vec{r}-\vec{r}\,'|}, \quad \text{in the direction } \hat{f} = f_x\hat{i}+f_y\hat{j}+f_z\hat{k} \qquad (1l)$$

Can simplify conceptually as the integral sum of a number of dipoles distributed through the volume of the body, analogous to gravity, but with two opposite-signed source terms, AND, having to consider field vector orientation (inclination and declination)!



Poisson's relation between Magnetic and Gravity Potentials

If $\vec{M}$ *is a constant vector*: $M_\alpha(n_x\hat{i}+n_y\hat{j}+n_z\hat{k})$, *then* $\vec{M}\cdot\vec{\nabla} = M_\alpha\frac{\partial}{\partial\alpha} = M_\alpha(n_x\frac{\partial}{\partial x}+n_y\frac{\partial}{\partial y}+n_z\frac{\partial}{\partial z})$, and

$$W_\alpha(\vec{r}) = -\frac{M_\alpha}{4\pi}\frac{\partial}{\partial\alpha}\iiint_V\frac{dV'}{|\vec{r}-\vec{r}'|}, \quad \textit{so that the net magnetic field}, \qquad (1k)$$

$$\Rightarrow \vec{H}_f(\vec{r}) = -\nabla_f W_\alpha = -\frac{M_\alpha}{4\pi}\frac{\partial^2}{\partial\alpha\,\partial f}\iiint_V\frac{dV'}{|\vec{r}-\vec{r}'|}, \quad \text{in } \textit{the direction} \quad \hat{f}=f_x\hat{i}+f_y\hat{j}+f_z\hat{k} \qquad (1l)$$

Poission's Relation between net magnetic Field, $\vec{H}$ **, and *the gravitational potential*,** U **:**

From *Gauss' Law for gravity*: $\vec{\nabla}\cdot\vec{g} = -4\pi G\Delta\rho$, *we had obtained*, $\vec{\nabla}^2 U = -4\pi G\Delta\rho \qquad (1m)$

So, $U(\vec{r}) = -G\iiint_V\frac{\Delta\rho}{|\vec{r}-\vec{r}'|}dV'$, *again*, *by Helmholtz's Theorem* $\qquad (1n)$

Gravity anomaly along the direction $\hat{\alpha}$,

$$g_\alpha(\vec{r}) = -\frac{\partial U}{\partial\alpha} = G\Delta\rho\frac{\partial}{\partial\alpha}\iiint_V\frac{dV'}{|\vec{r}-\vec{r}'|} \qquad (1o)$$

Dividing $(1k)$ *by* $(1o)$:

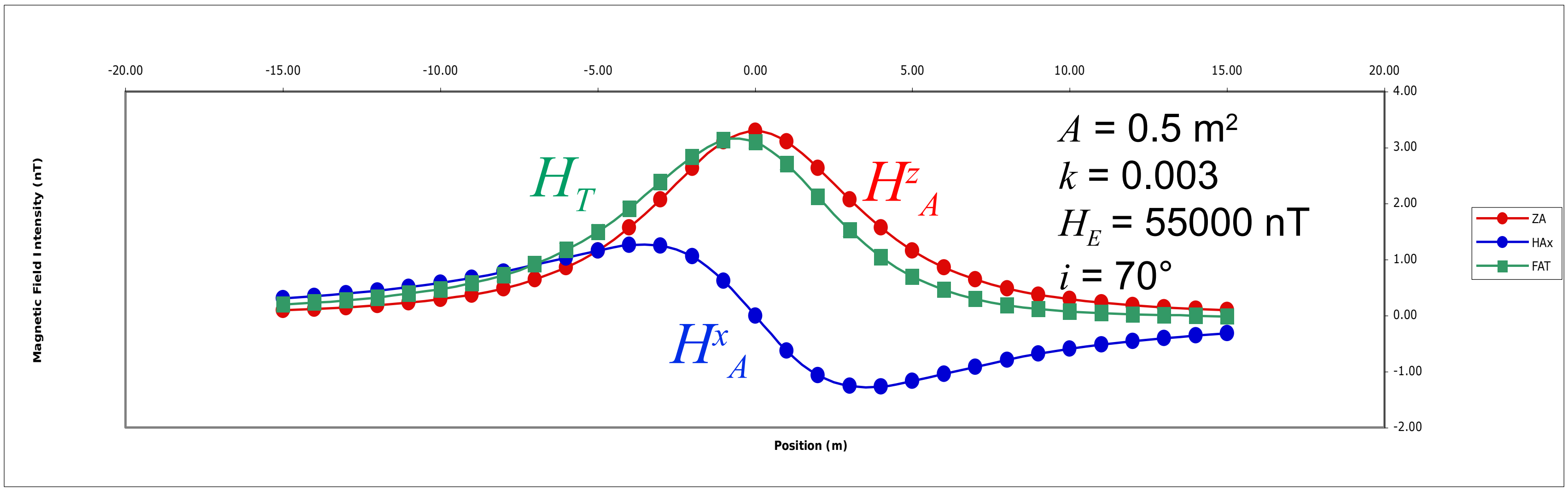
$$W_\alpha = -\left(\frac{M_\alpha}{4\pi G\Delta\rho}\right)g_\alpha = -\left(\frac{M_\alpha}{4\pi G\Delta\rho}\right)\frac{\partial U}{\partial\alpha}, \quad \textit{so that} \qquad (1p)$$

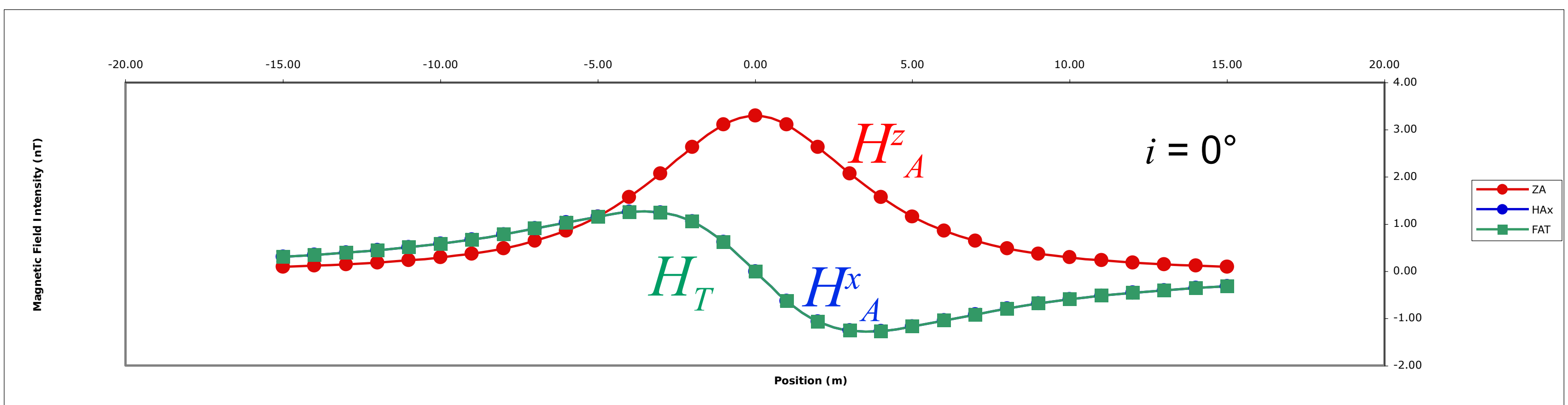
$$\vec{H}_f = -\nabla_f W_\alpha = \left(\frac{M_\alpha}{4\pi G\Delta\rho}\right)\frac{\partial^2 U}{\partial\alpha\,\partial f} \qquad (\textbf{POISSON's RELATION}) \qquad (1q)$$

We define the x direction as toward magnetic north, and the total-field ***magnetic anomaly*** (difference from the magnitude of the main field) is a scalar that can be expressed as: (i is inclination)

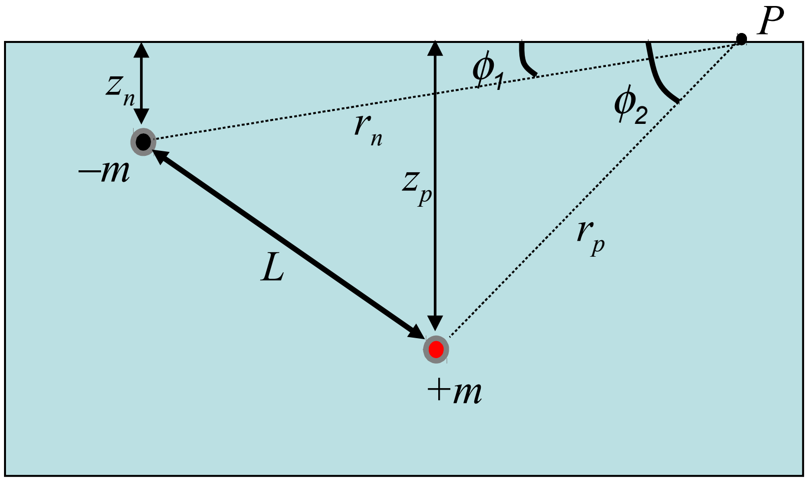
$$H_T = H_A^x \cos i + H_A^z \sin i$$

$$\vec{H}_A = \hat{x}\frac{xkH_E A}{r^3} + \hat{y}\frac{ykH_E A}{r^3} + \hat{z}\frac{zkH_E A}{r^3}$$

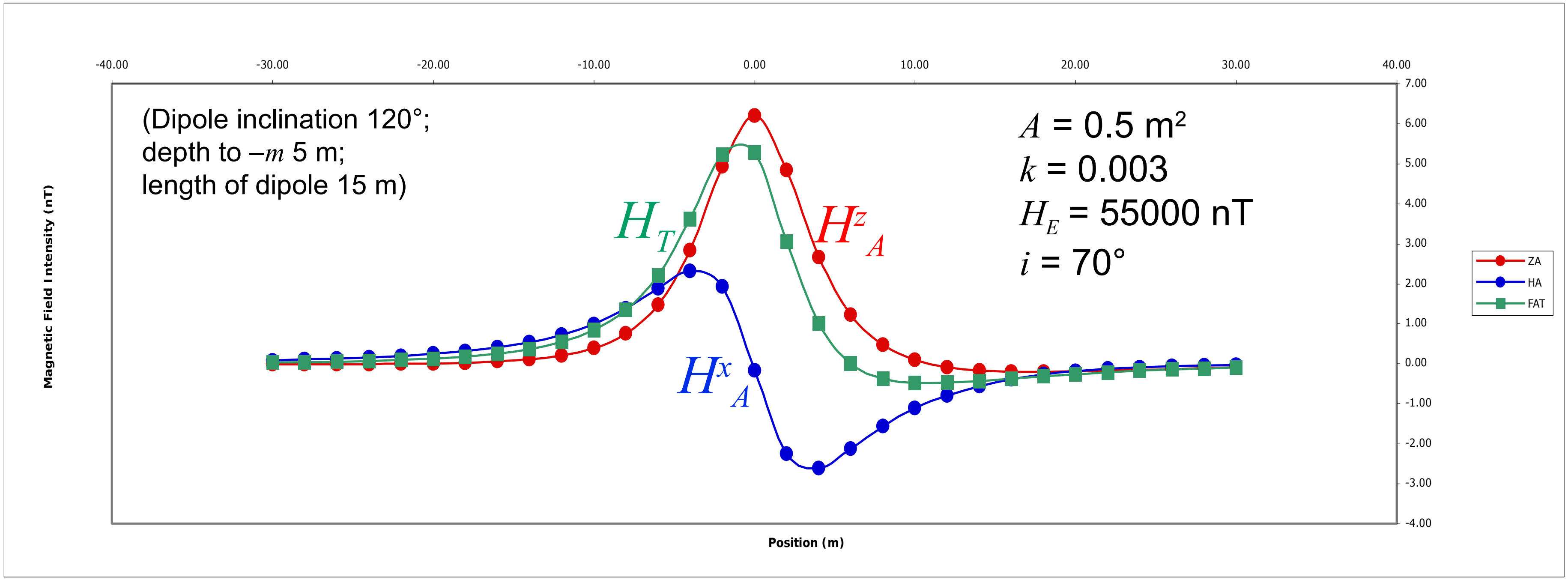


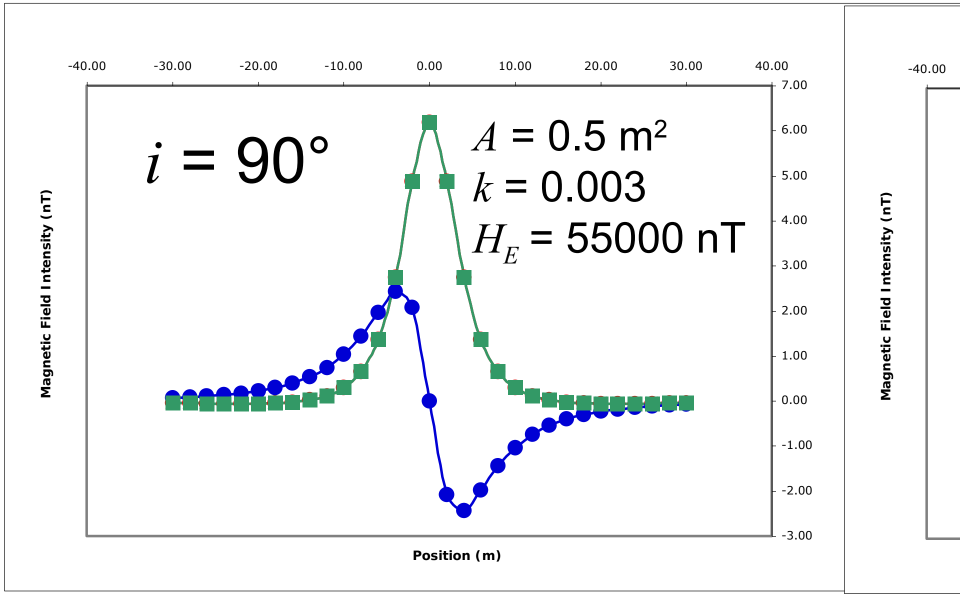


For a ***dipole***, the magnetic field is the sum of anomalies generated by the positive and negative "monopoles":

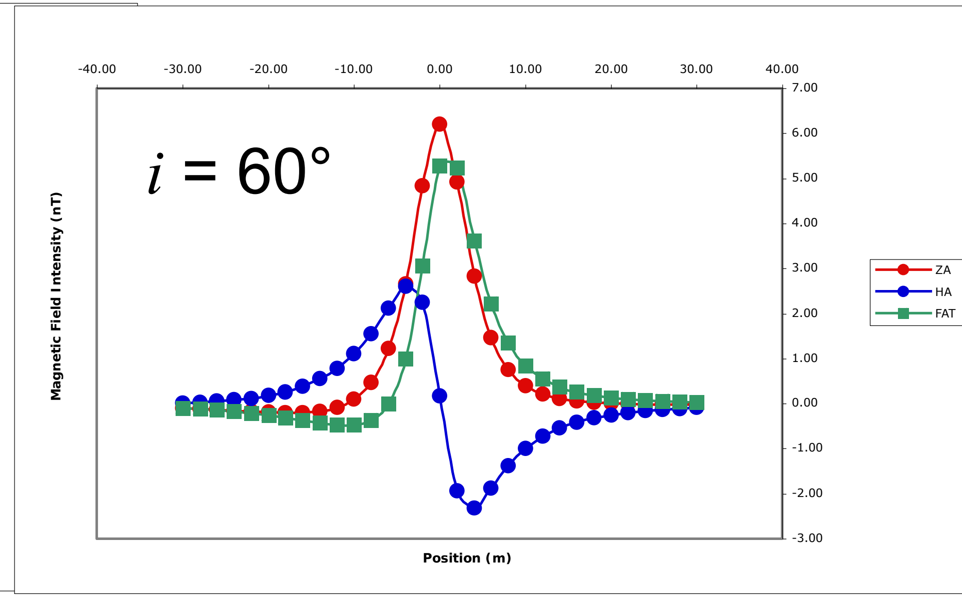


$$\vec{H}_a = kH_E A\left[\hat{x}\left(\frac{\cos\phi_1}{r_n^2} - \frac{\cos\phi_2}{r_p^2}\right) + \hat{z}\left(\frac{\sin\phi_1}{r_n^2} - \frac{\sin\phi_2}{r_p^2}\right)\right]$$

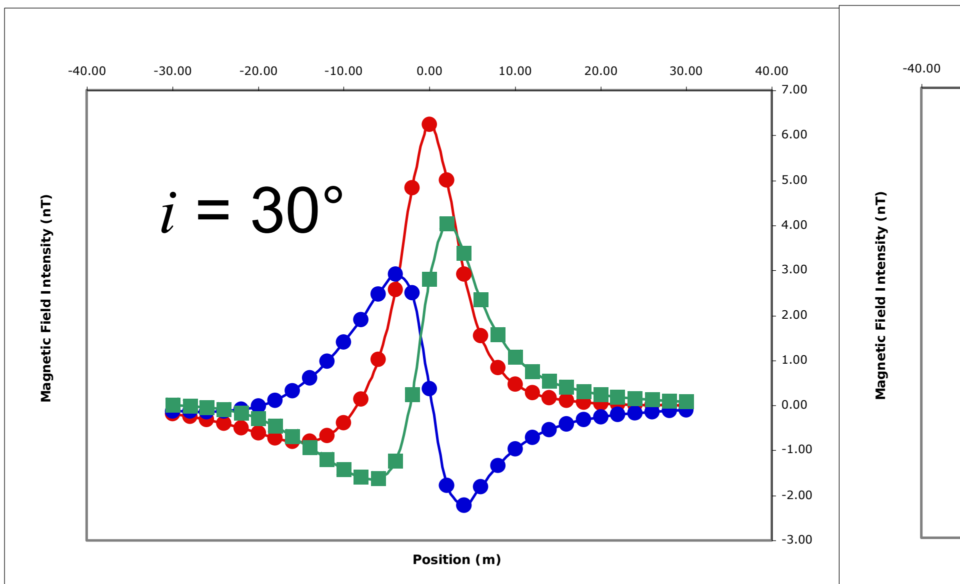




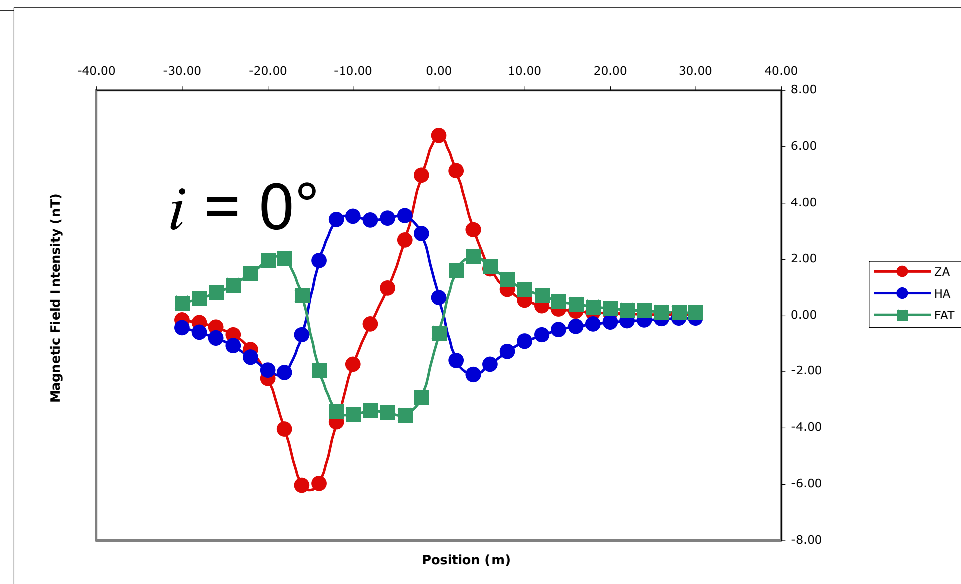
i = 90°
A = 0.5 m²
k = 0.003
H_E = 55000 nT
Magnetic Field Intensity (nT)
Position (m)
-40.00 -30.00 -20.00 -10.00 0.00 10.00 20.00 30.00 40.00
7.00 6.00 5.00 4.00 3.00 2.00 1.00 0.00 -1.00 -2.00 -3.00



i = 60°
Magnetic Field Intensity (nT)
Position (m)
-40.00 -30.00 -20.00 -10.00 0.00 10.00 20.00 30.00 40.00
7.00 6.00 5.00 4.00 3.00 2.00 1.00 0.00 -1.00 -2.00 -3.00
ZA
HA
FAT



i = 30°
Magnetic Field Intensity (nT)
Position (m)
-40.00 -30.00 -20.00 -10.00 0.00 10.00 20.00 30.00 40.00
7.00 6.00 5.00 4.00 3.00 2.00 1.00 0.00 -1.00 -2.00 -3.00



i = 0°
Magnetic Field Intensity (nT)
Position (m)
-40.00 -30.00 -20.00 -10.00 0.00 10.00 20.00 30.00 40.00
8.00 6.00 4.00 2.00 0.00 -2.00 -4.00 -6.00 -8.00
ZA
HA
FAT

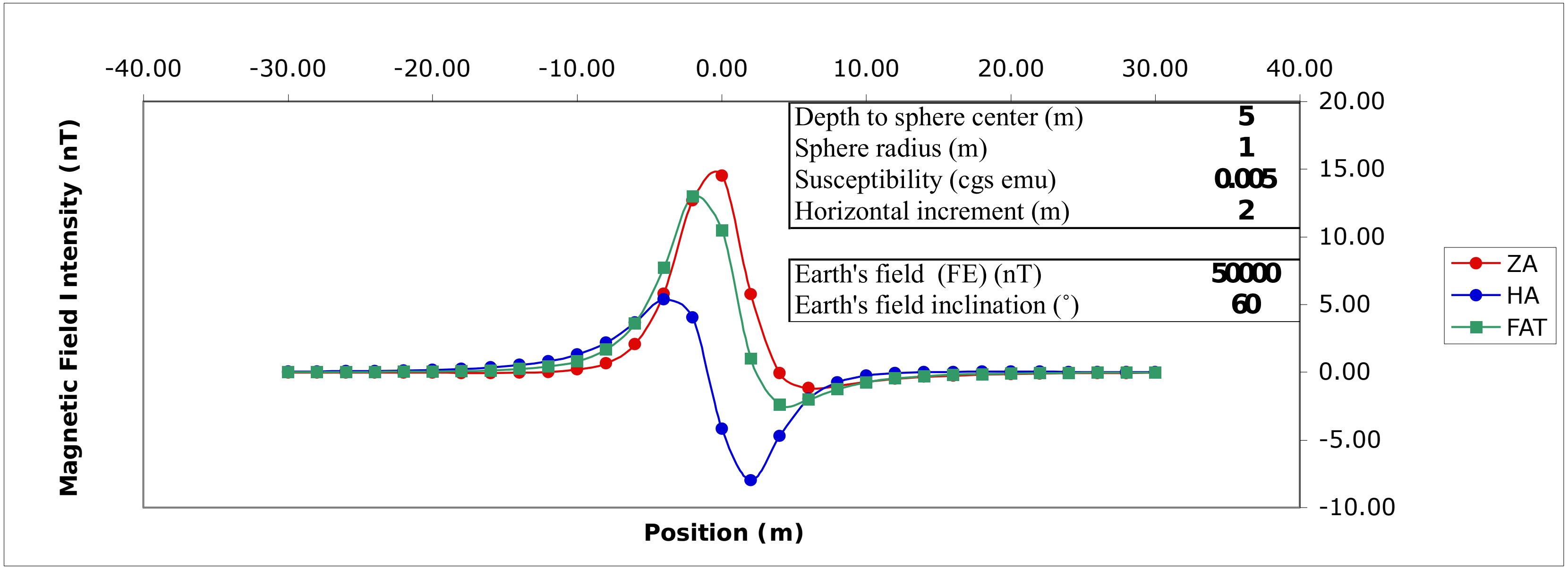
Example:

Anomaly due to a sphere of radius R has vertical component

$$H_a^z = \frac{4\pi R^3 k H_E \sin i}{3\left(x^2+z^2\right)^{3/2}}\left[\frac{3z^2 - 3xz\cot i}{x^2+z^2} - 1\right]$$

and horizontal component

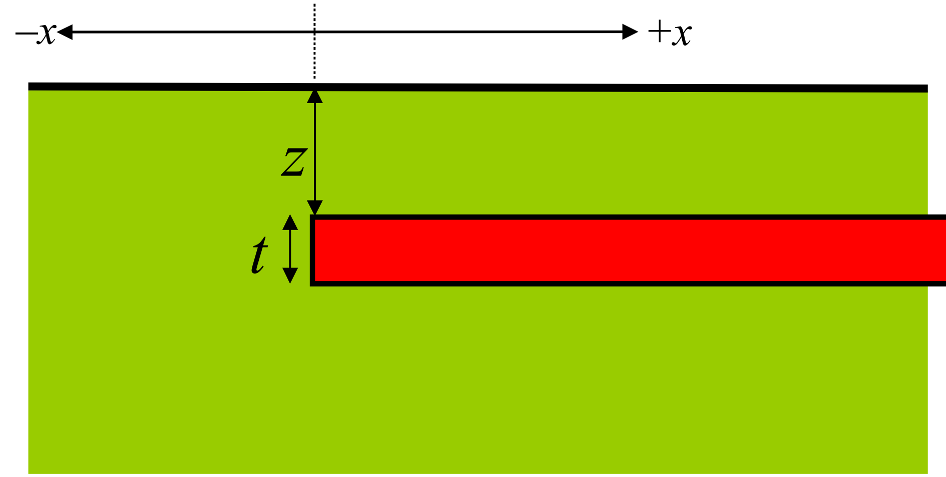
$$H_a^x = \frac{4\pi R^3 k H_E \cos i}{3\left(x^2+z^2\right)^{3/2}}\left[\frac{3x^2 - 3xz\tan i}{x^2+z^2} - 1\right]$$

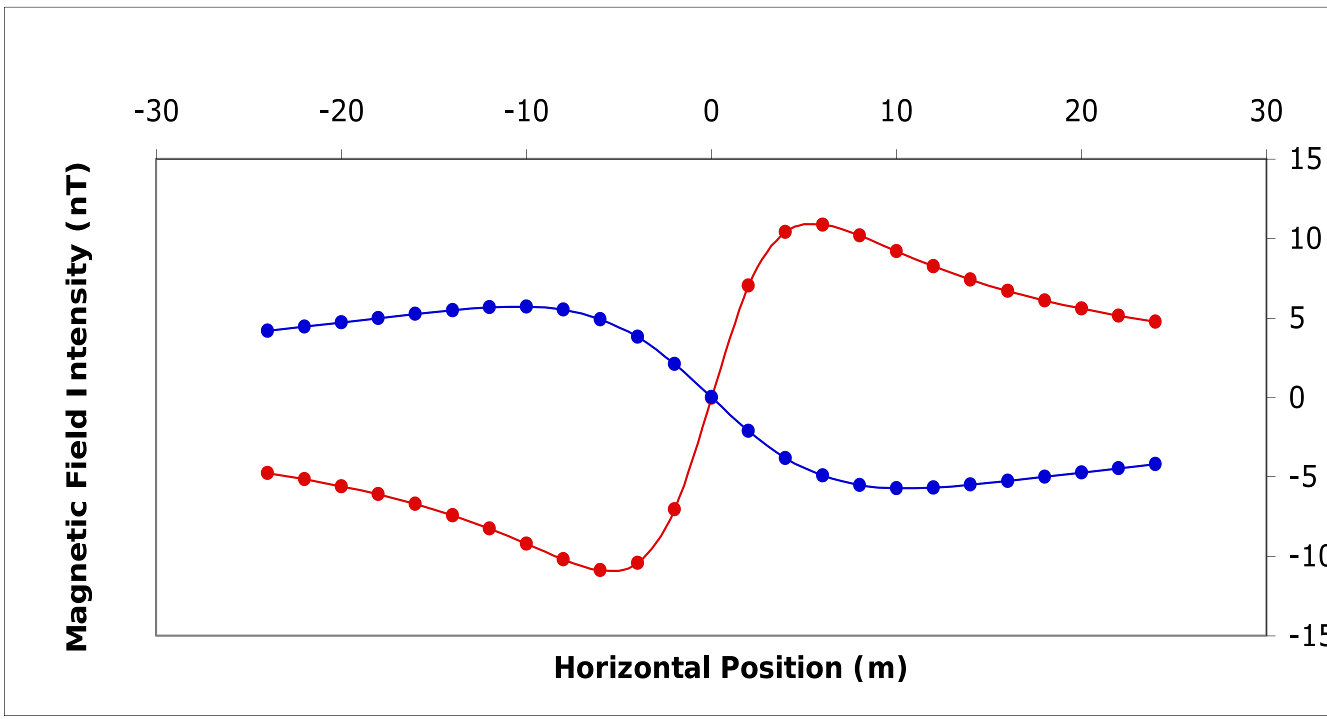


Anomaly due to a semi-infinite sheet has vertical component

$$H_a^z = 2kH_E\left[\tan^{-1}\left(\frac{x}{z}\right) - \tan^{-1}\left(\frac{x}{z+t}\right)\right]$$

for vertical Earth field… More algebra if more general…

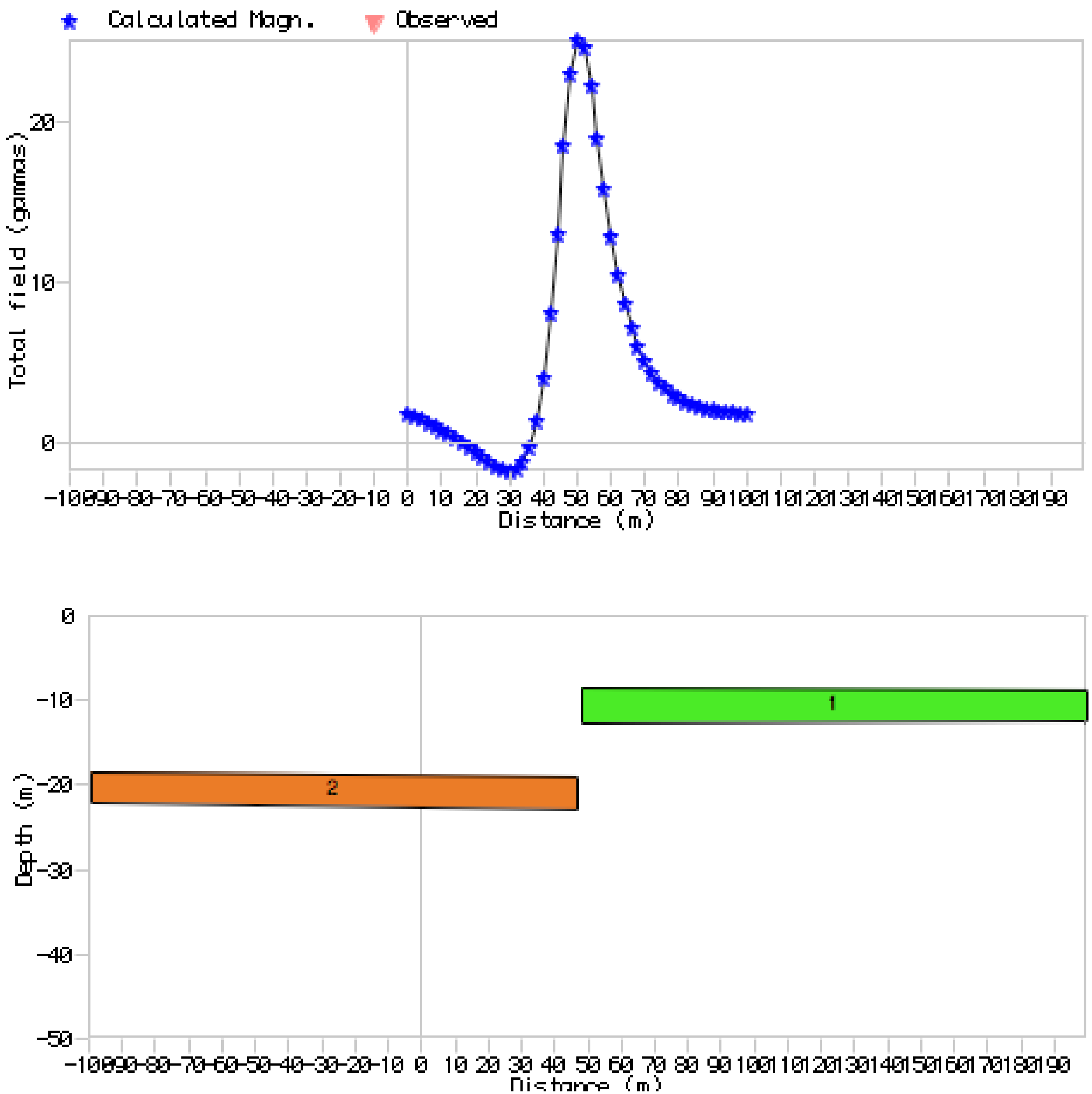




	Sheet A
Depth to sheet (m)	**5**
Sheet thickness (m)	**1**
Susceptibility (emu)	**0.001**
Horizontal increment (m)	**2**

	Sheet B
Depth to sheet (m)	**10**
Sheet thickness (m)	**1**
Susceptibility (emu)	**0.001**
Horizontal increment (m)	**2**

Note in GravMag, you must enter "Preferences" in order to specify inclination/ declination of the Earth's main field (then the program allows you to input azimuth of the profile relative to geographical north, which you will need to calculate from GPS positions).



Also does remanent magnetization, which we neglect here.