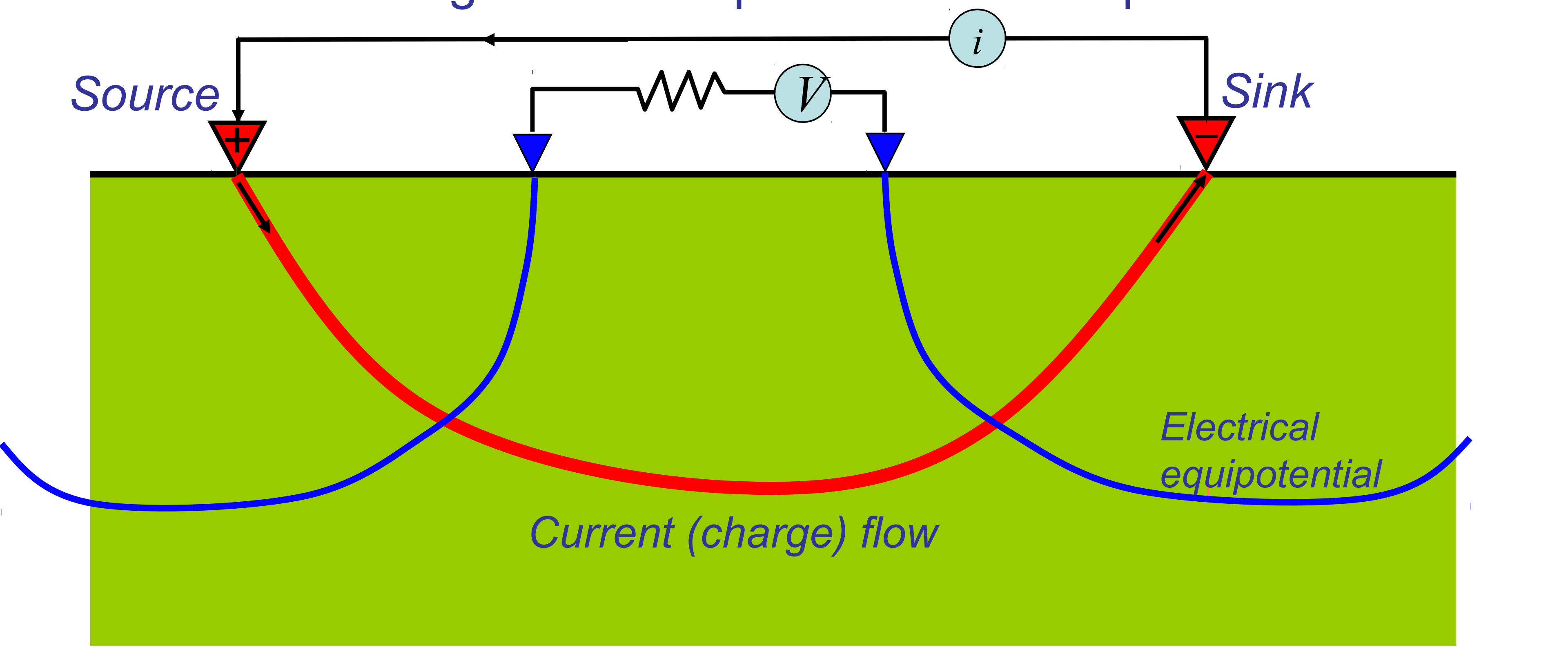
DC Electrical Resistivity

Method:

- Apply a direct current (or very low frequency alternating current) to the Earth using a dipole current source/sink
- Measure voltage across a pair of electrode probes



The measured voltage represents a difference in potential…

Advantage: Instrumentation doesn't require great sensitivity!

DC Electrical Resistivity

Similar to gravity and magnetics, DC resistivity is a ***potential field method***:

Maxwell's Equations & Lorentz Force:

$$\nabla\cdot\vec{B} = 0, \qquad \textit{Gauss' Law for Magnetism} \qquad (1a)$$

$$\nabla\cdot\vec{H} = 0, \qquad \textit{Gauss' Law Variation } [\textit{under the assumption: } \vec{H}\approx\vec{B}, \textit{ when } (\mu M_{crust}) \ll B] \qquad (1a')$$

$$\nabla\cdot\vec{E} = \frac{\sigma}{\epsilon}, \qquad \textit{Gauss' Law for Electrostatics} \qquad (1b)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \qquad \text{Faraday-Maxwell Law} \qquad (1c)$$

$$\vec{\nabla}\times\vec{B} = \mu\left[\vec{J} + \epsilon\frac{\partial\vec{E}}{\partial t}\right], \quad \text{Ampere-Maxwell Law,} \qquad \text{where } \vec{J} = \frac{d\vec{i}}{dA} = \sigma\vec{E} = \frac{\vec{E}}{\rho} \quad \text{(Ohm's Law)} \qquad (1d)$$

$$\vec{F} = q[\vec{E} + q(\vec{v}\times\vec{B})], \qquad \text{Lorentz Force} \qquad (1e)$$

Poisson's Equation for Electrostatics:

Assuming no time-variation in electric and magnetic fields, i.e., $\frac{d\vec{B}}{dt} = 0 = \frac{d\vec{E}}{dt}$, we get, $\nabla\times\vec{E}=0$.

Thus, there exists a scalar potential, V, such that $\vec{E}=\nabla V$, and therefore, by Gauss' Law (1b):

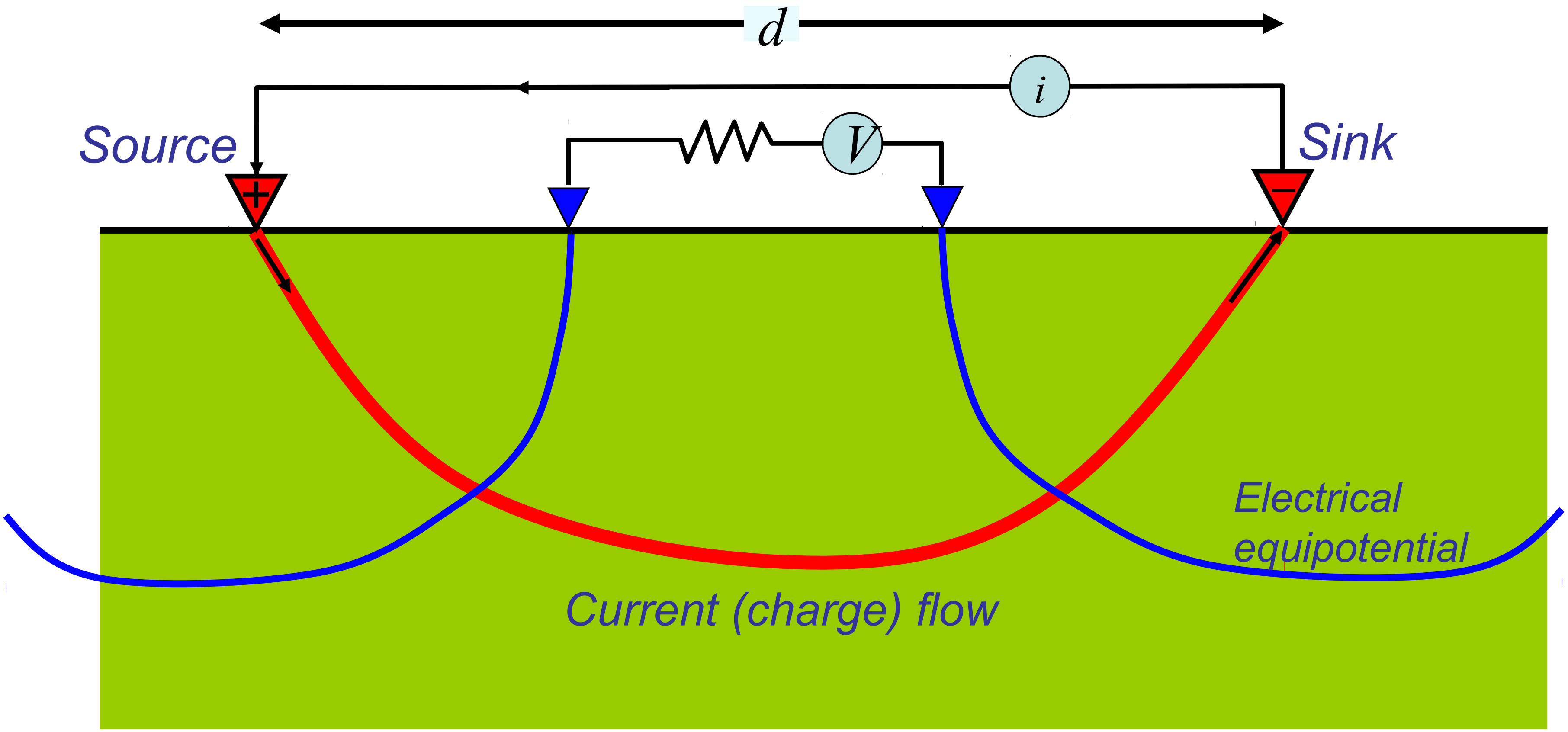
$$\nabla^2 V = \frac{\sigma}{\epsilon} \qquad \text{(Poisson's Equation)} \qquad (1f)$$

Laplace's Equation for Electrostatics:

If we further assume no significant free charge distribution within the crust, $\sigma=0$, we obtain:

$$\nabla^2 V = 0 \qquad \text{(Laplace's Equation)} \qquad (1g)$$

DC Electrical Resistivity

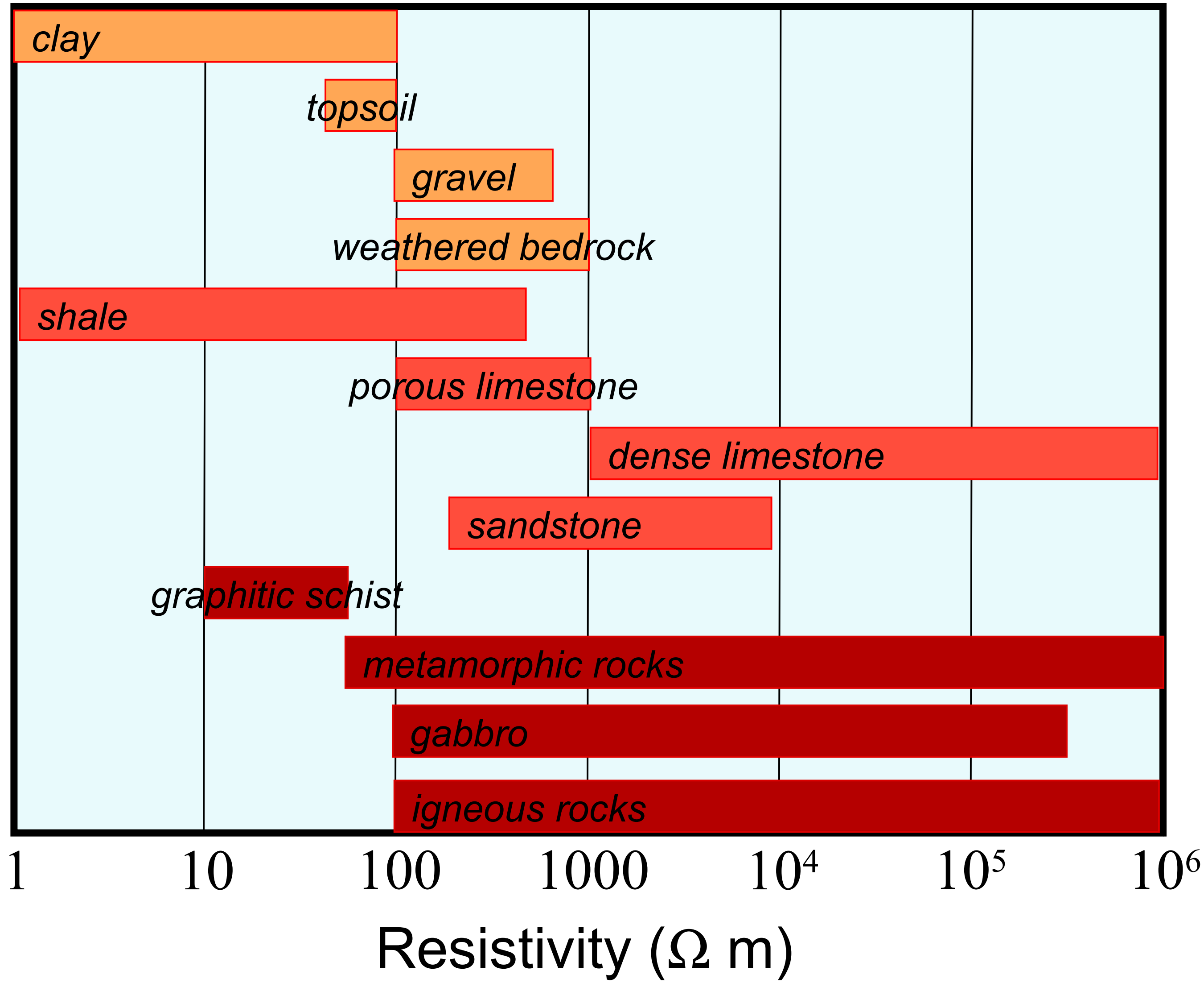


Ohm's Law:

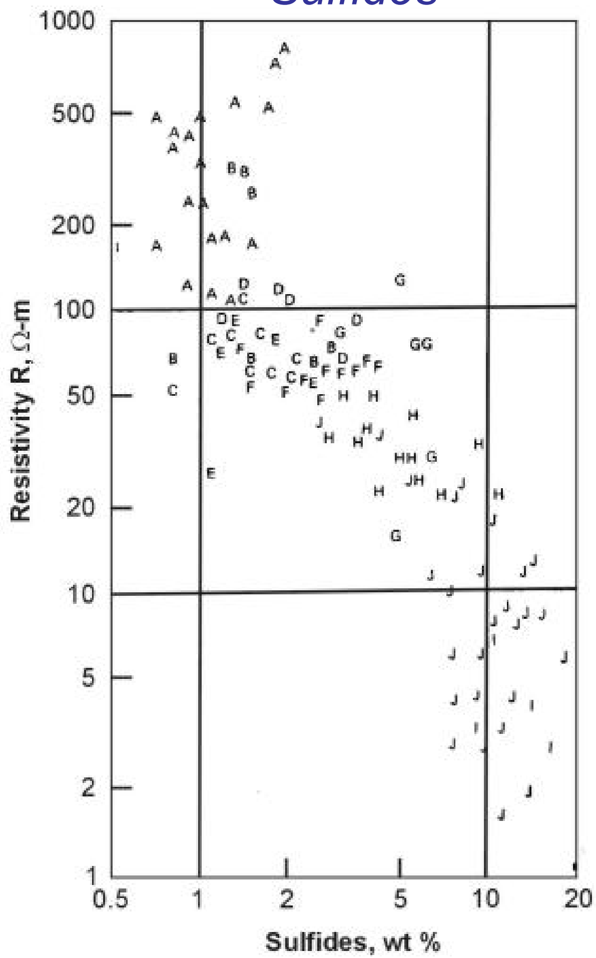
$$\vec{J} = \sigma\vec{E} = \frac{\vec{E}}{\rho} = -\frac{\vec{\nabla}V}{\rho}$$

So for a given current i and electrode spacing d (defines $\vec{J}$), V depends on ρ

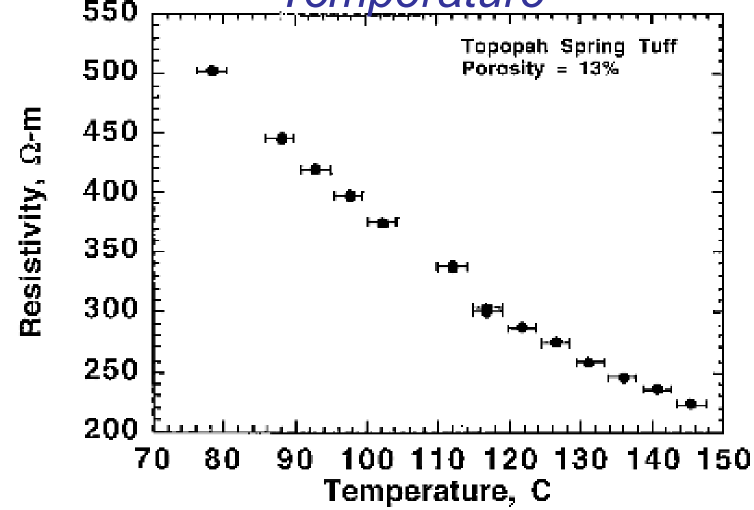
Resistivity properties of rocks & soils:

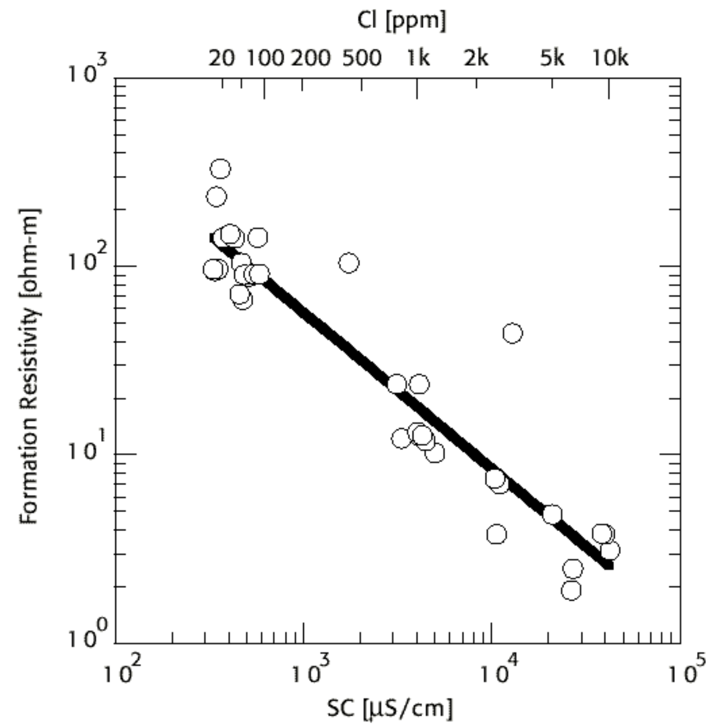


Sulfides



Temperature





Water Salinity in Pore Fluids

Example: Current source I in an infinite homogeneous medium with constant resistivity ρ

We expect $V = 0$ at $r = \infty$, &
equipotential surfaces to be spherical around the source (similar to gravity!)

So, by Ohm's Law: $$\vec{J} = -\sigma \vec{\nabla} V = -\sigma \frac{\partial V}{\partial r} \hat{r} = \frac{\sigma A}{r^2} \hat{r}$$

for some unknown, *A*. We can express *A* in terms of the ground current,

$$\vec{I} \equiv 4\pi r^2 \vec{J} = 4\pi\sigma A \hat{r}$$

so that, $$A = \frac{I}{4\pi\sigma} = \frac{I\rho}{4\pi}$$

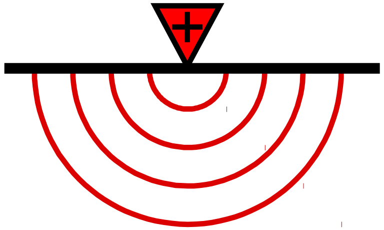
Now, integrating $\partial V/\partial r = A/r^2 = I\rho/(4\pi r^2)$, we finally obtain:

$$V = \frac{I\rho}{4\pi r}$$ or, equivalently: $$\rho = \frac{4\pi r V}{I}$$

So given a known I & a measured voltage at known r, we can solve for a constant resistivity ρ.

More realistic representation of Earth is a halfspace:

$$\left.\frac{\partial V}{\partial z}\right|_{z=0} = 0$$

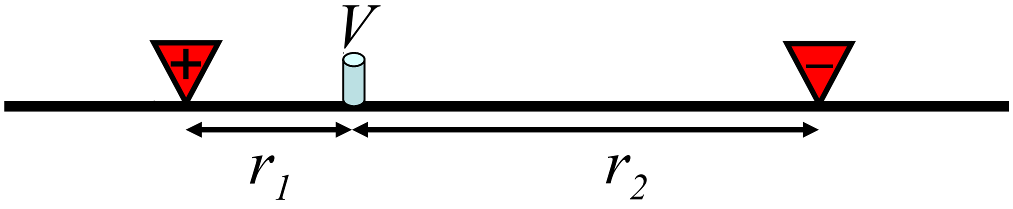


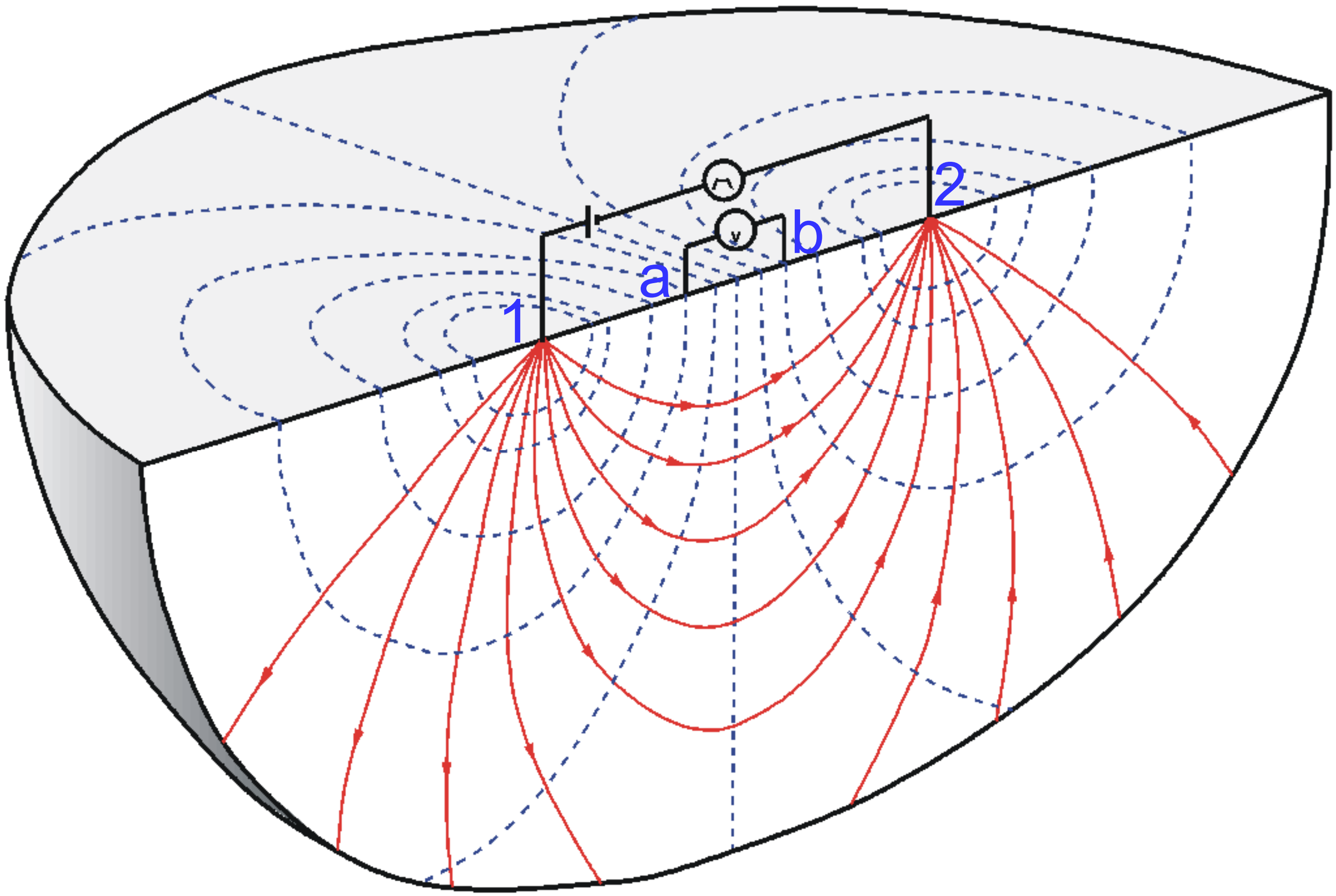
Same current is forced into half the volume so

$$V = \frac{I\rho}{2\pi r} \quad \text{or equivalently} \quad \rho = \frac{2\pi r V}{I}$$

And for two current electrodes $+I$ and $-I$, total potential is given by the sum of the two point sources

$$V = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi}\left(\frac{r_2 - r_1}{r_2 r_1}\right)$$





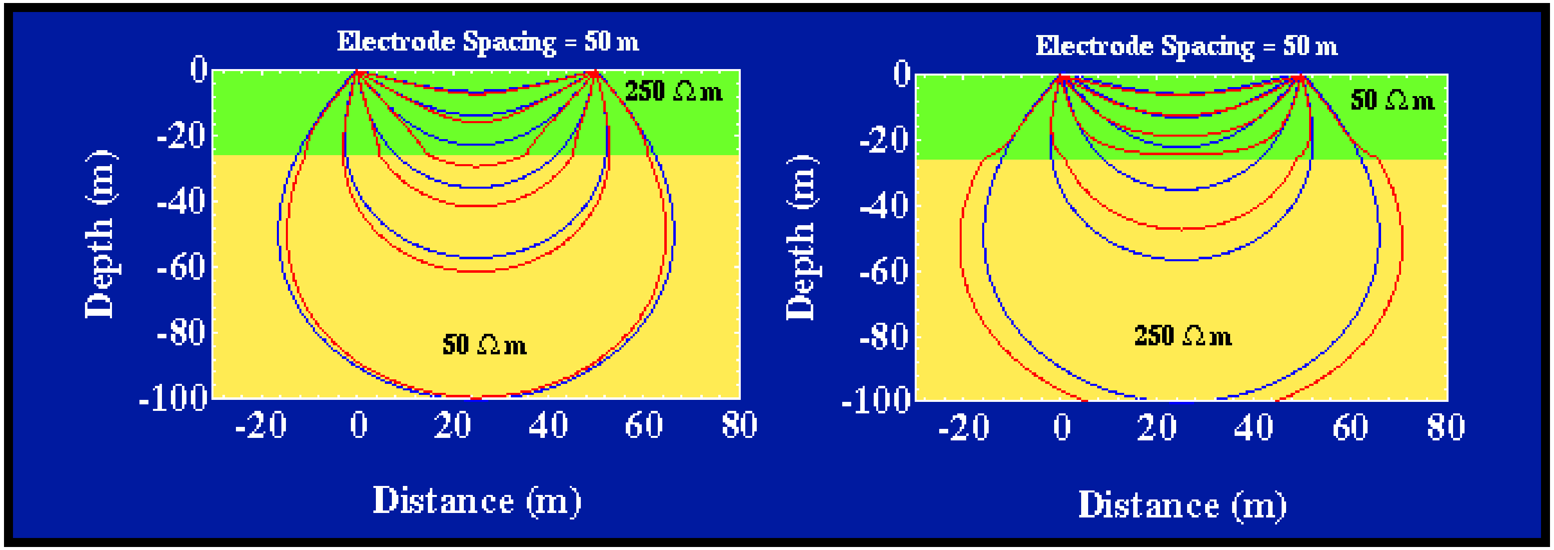
In practice we measure voltage difference at two points,

$$\Delta V = V_a - V_b$$

$$\Delta V = \frac{I\rho}{2\pi}\left(\frac{r_{2a} - r_{1a}}{r_{2a}r_{1a}} - \frac{r_{2b} - r_{1b}}{r_{2b}r_{1b}}\right)$$

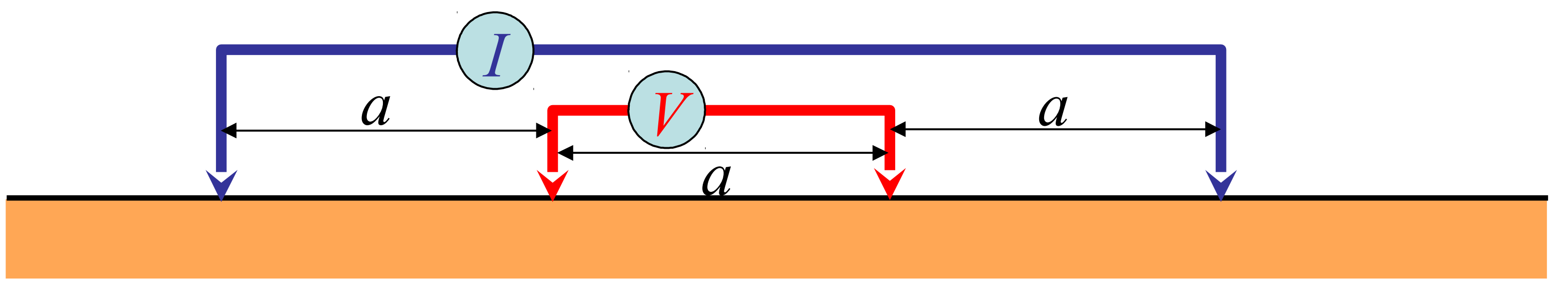
$$\rho_{app} = \frac{2\pi\Delta V}{I}\left(\frac{1}{\frac{1}{r_{1a}} - \frac{1}{r_{1b}} - \frac{1}{r_{2a}} + \frac{1}{r_{2b}}}\right)$$

We are really interested however in imaging a subsurface in which ρ varies. The potentials integrate ρ over the volume so they provide information relevant for doing exactly that!



Electrode Arrays: Arrangement of the electrodes

Wenner Array: (Classical: common in older surveys)

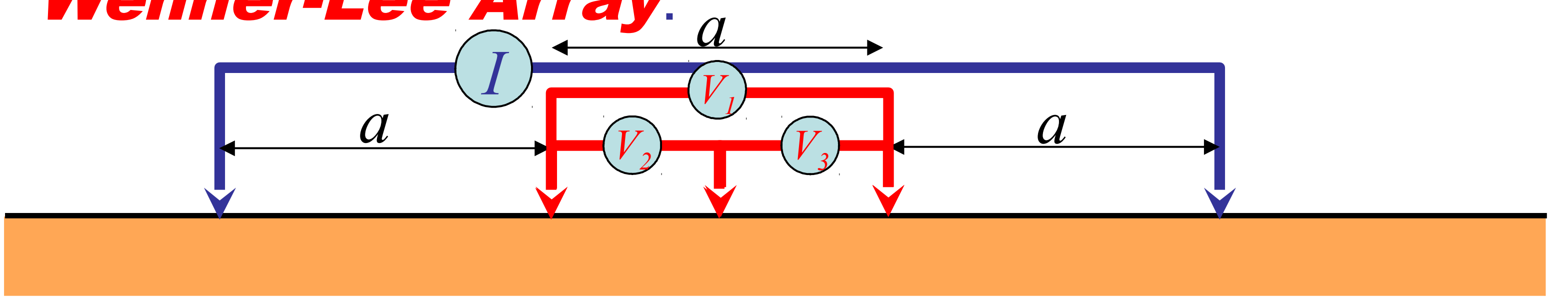


Constant spacing ("*a*-spacing"):

$$r_{1a} = r_{2b} = a \qquad r_{1b} = r_{2a} = 2a$$

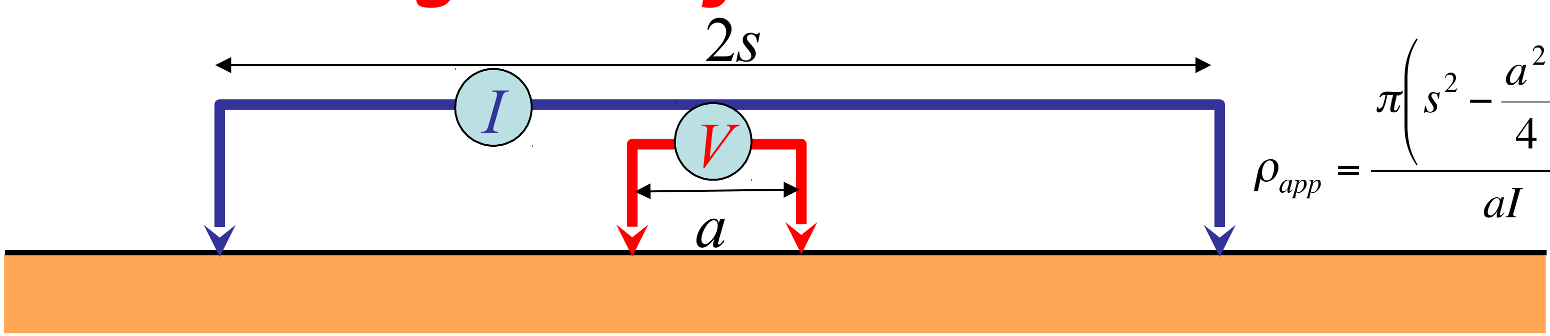
$$\rho_{app} = \frac{2\pi\Delta V}{I}\left(\frac{1}{\frac{1}{r_{1a}} - \frac{1}{r_{1b}} - \frac{1}{r_{2a}} + \frac{1}{r_{2b}}}\right) = \frac{2\pi a\Delta V}{I}$$

Wenner-Lee Array:



Additional voltage measurements to a center electrode reduce sensitivity to near-surface resistivity variations

Schlumberger Array:



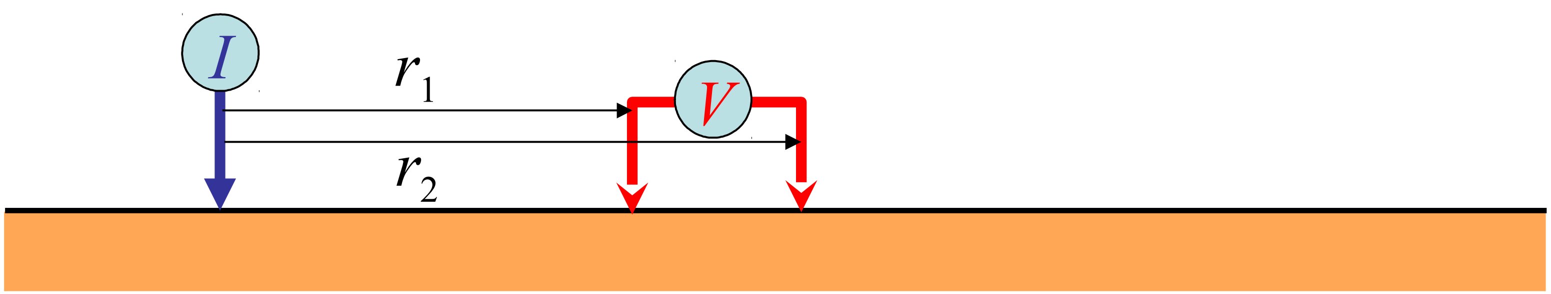
$$\rho_{app} = \frac{\pi\left(s^2 - \frac{a^2}{4}\right)\Delta V}{aI}$$

Also less sensitive to near-surface resistivity variations, & required half the effort to move electrodes for ***sounding*** studies (of vertical changes in resistivity)

Dipole-Dipole Array:

Requires larger current source, but fairly common now with the development of multichannel instruments

Pole-Dipole Array:



Most sensitive to resistivity in the shell between radii r_1 & r_2 – commonly used for tunnel/karst detection

Halfspace Current Distribution

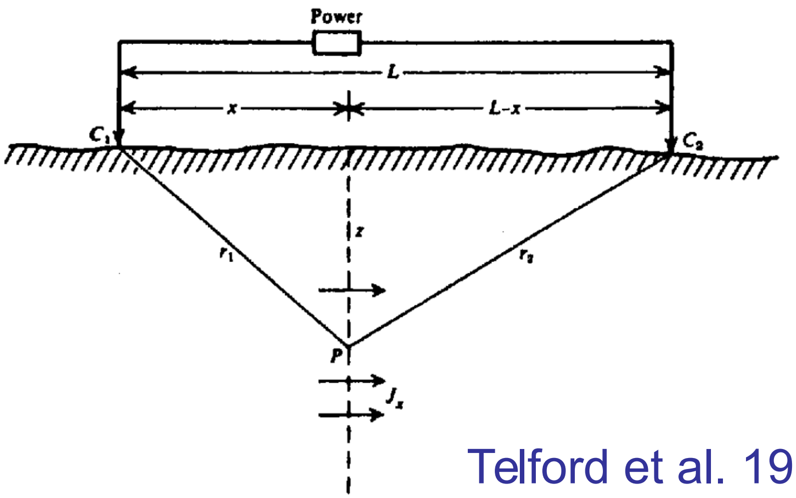
$$\textit{Current Density},\ \vec{J} = \sigma\vec{E} = \frac{\vec{E}}{\rho} = -\frac{\vec{\nabla} V}{\rho} \qquad (1a)$$

Horizontal current density at point P due to point electrodes at C_1 and C_2 is

$$\begin{aligned} J_x &= -\left(\frac{1}{\rho}\right)\frac{\partial V}{\partial x} \\ &= -\left(\frac{I}{2\pi}\right)\frac{\partial}{\partial x}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \\ &= -\left(\frac{I}{2\pi}\right)\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}-\frac{1}{\sqrt{(L-x)^2+y^2+z^2}}\right) \\ &= \left(\frac{I}{2\pi}\right)\left(\frac{x}{\sqrt{(x^2+y^2+z^2)^3}}+\frac{(L-x)}{\sqrt{[(L-x)^2+y^2+z^2]^3}}\right) \end{aligned} \qquad (1b)$$

If this point were on a plane equidistant from electrodes C_1 and C_2, then $x=L/2=(L-x)$, *then*

$$J_x = \left(\frac{I}{2\pi}\right)\frac{L}{\sqrt{\left(\frac{L^2}{4}+y^2+z^2\right)^3}} \qquad (1c)$$



Telford et al. 1990

Figure 8.5. Determining the current density in uniform ground below two surface electrodes.

So, the fraction of current flowing below a depth of z_0(to ∞) would be,

$$dI_x = \left(\frac{I}{2\pi}\right)\frac{L}{\sqrt{\left(\frac{L^2}{4}+y^2+z^2\right)^3}}dydz \tag{1d}$$

So,

$$\frac{I_x}{I} = \left(\frac{L}{2\pi}\right)\int_{z=z0}^{\infty} dz\,.\int_{y=-\infty}^{\infty}\frac{dy}{\sqrt{\left(\frac{L^2}{4}+y^2+z^2\right)^3}}$$

$$= \left(\frac{L}{2\pi}\right)\int_{z=z0}^{\infty} dz\,.\left|\frac{y}{\left(\frac{L^2}{4}+z^2\right)\sqrt{\left(\frac{L^2}{4}+y^2+z^2\right)}}\right|_{-\infty}^{\infty}$$

$$= \left(\frac{L}{2\pi}\right)\int_{z=z0}^{\infty}\frac{2\,dz}{\left(\frac{L^2}{4}+z^2\right)}$$

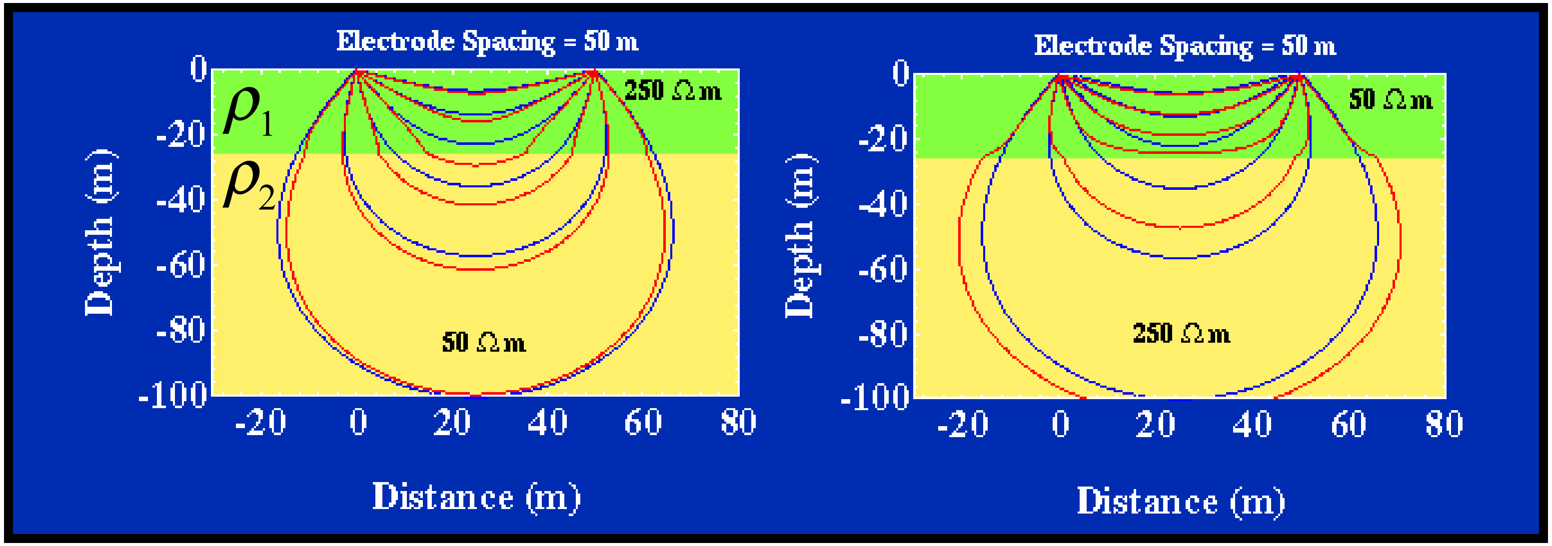
$$= \left(\frac{L}{\pi}\right)\left|\frac{2}{L}\tan^{-1}\left(\frac{2z}{L}\right)\right|_{z_0}^{\infty}$$

Thus,

$$\frac{I_x}{I} = \frac{2}{\pi}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2z_0}{L}\right)\right] \tag{1e}$$

Modeling

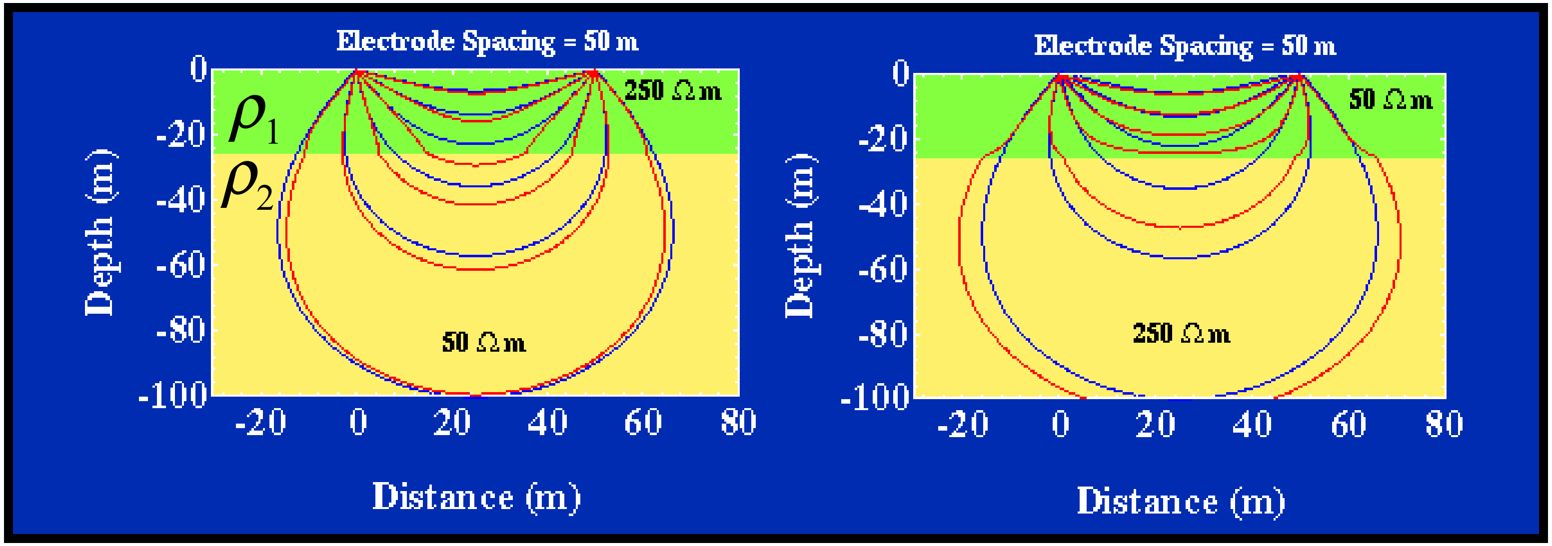
Current penetration for a **layer over half-space**:

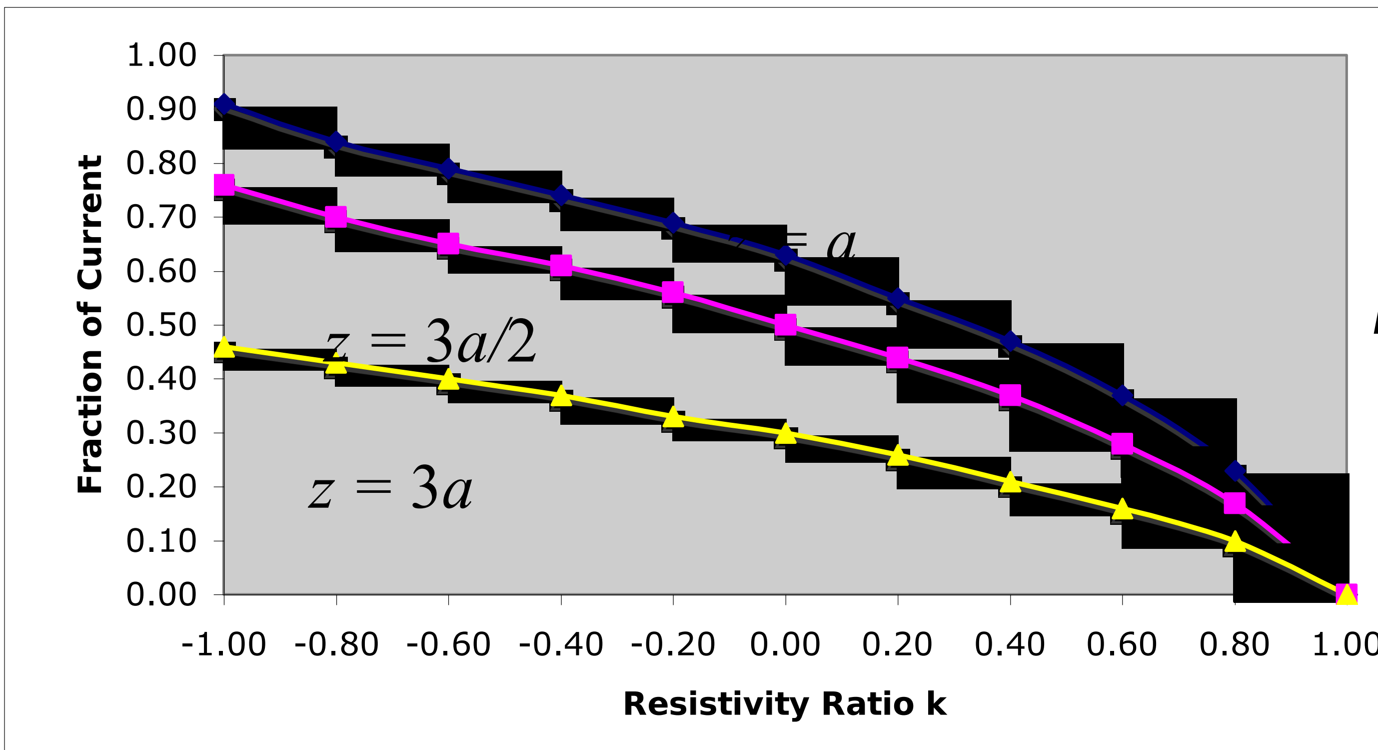


Current penetration into the second layer depends on depth of layer interface, & resistivities. Fraction of total current that penetrates below depth of an interface, z, is (for a Wenner array):

$$\frac{I}{I_o} = \frac{2\rho_1}{\pi\rho_2}(k+1)\sum_{n=0}^{\infty} k^n \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{2(2n+1)z}{3a}\right)\right]$$

where a is electrode spacing, and $k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$

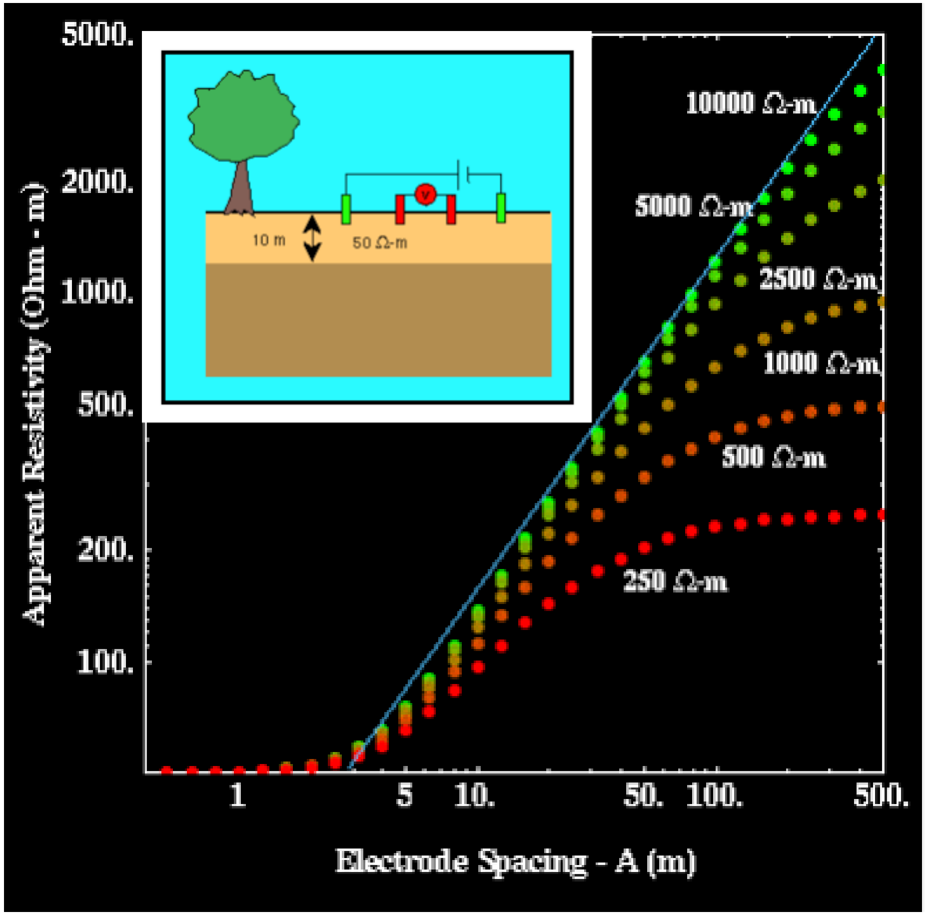




$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

Layer over half-space:

Apparent resistivity for a Wenner array:

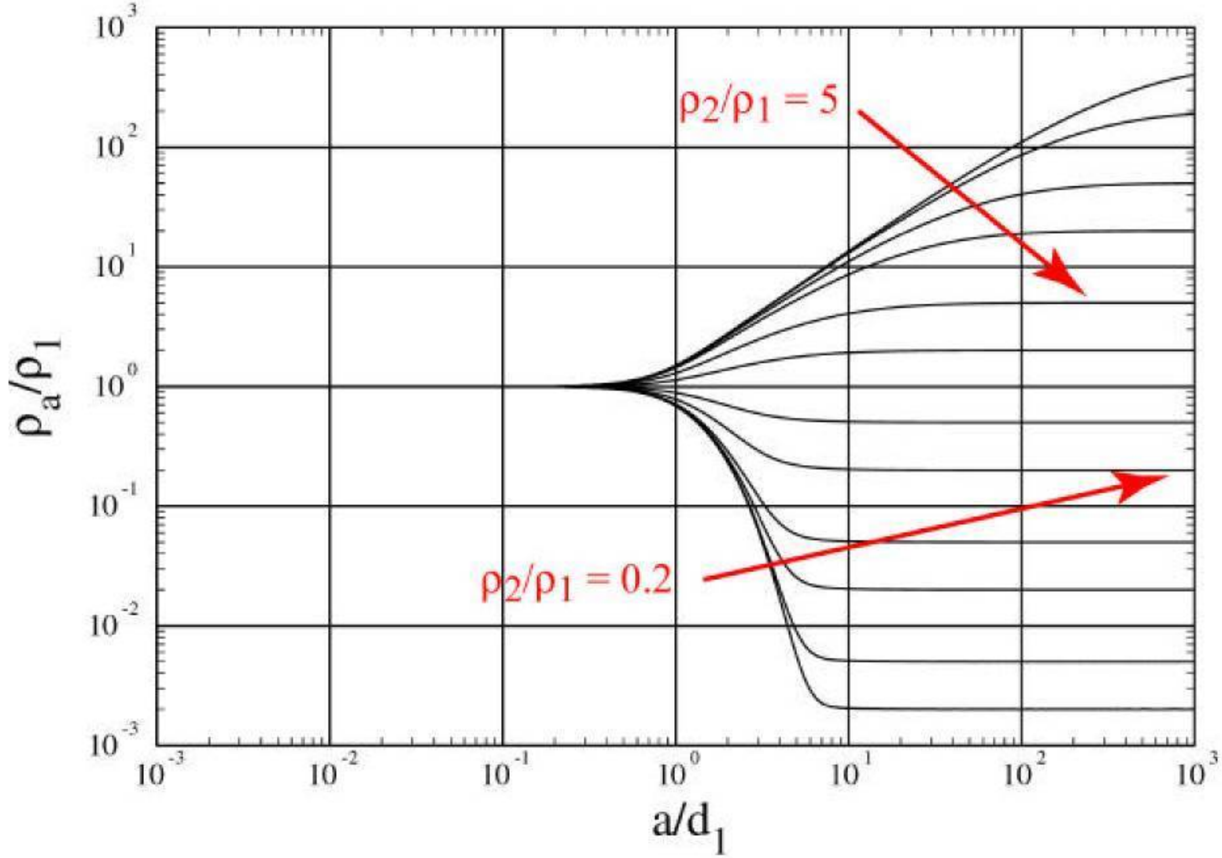


$$\rho_{app} = \rho_1 \left[1 + 4\sum_{n=1}^{\infty} k^n \sqrt{1 + \left(\frac{2nz}{a}\right)^2} - 2\sum_{n=1}^{\infty} \frac{k^n}{\sqrt{1 + (nz/a)^2}} \right]$$

$\rho_{app} \sim \rho_1$ for a/z small

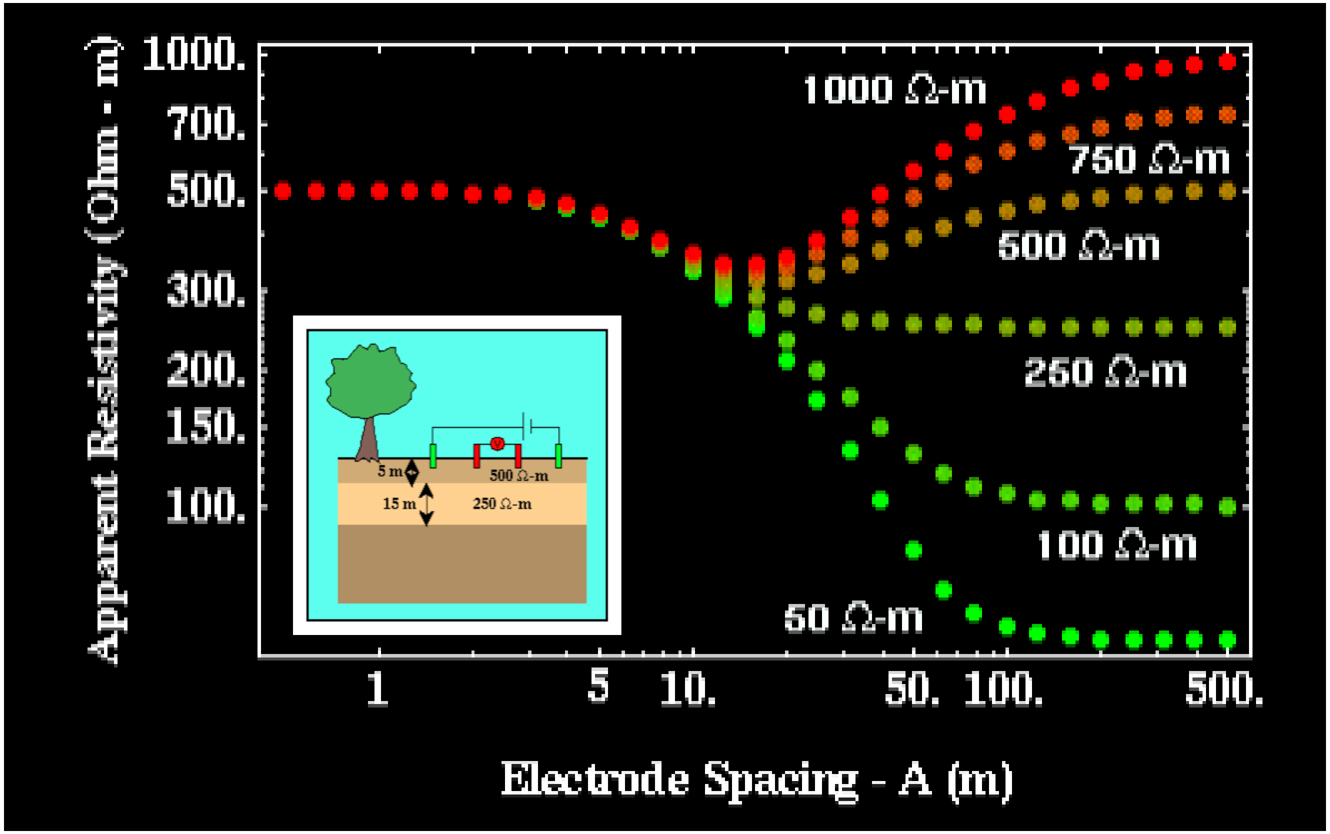
$\sim \rho_2$ for a/z large

Wenner Array: “Universal Curves"

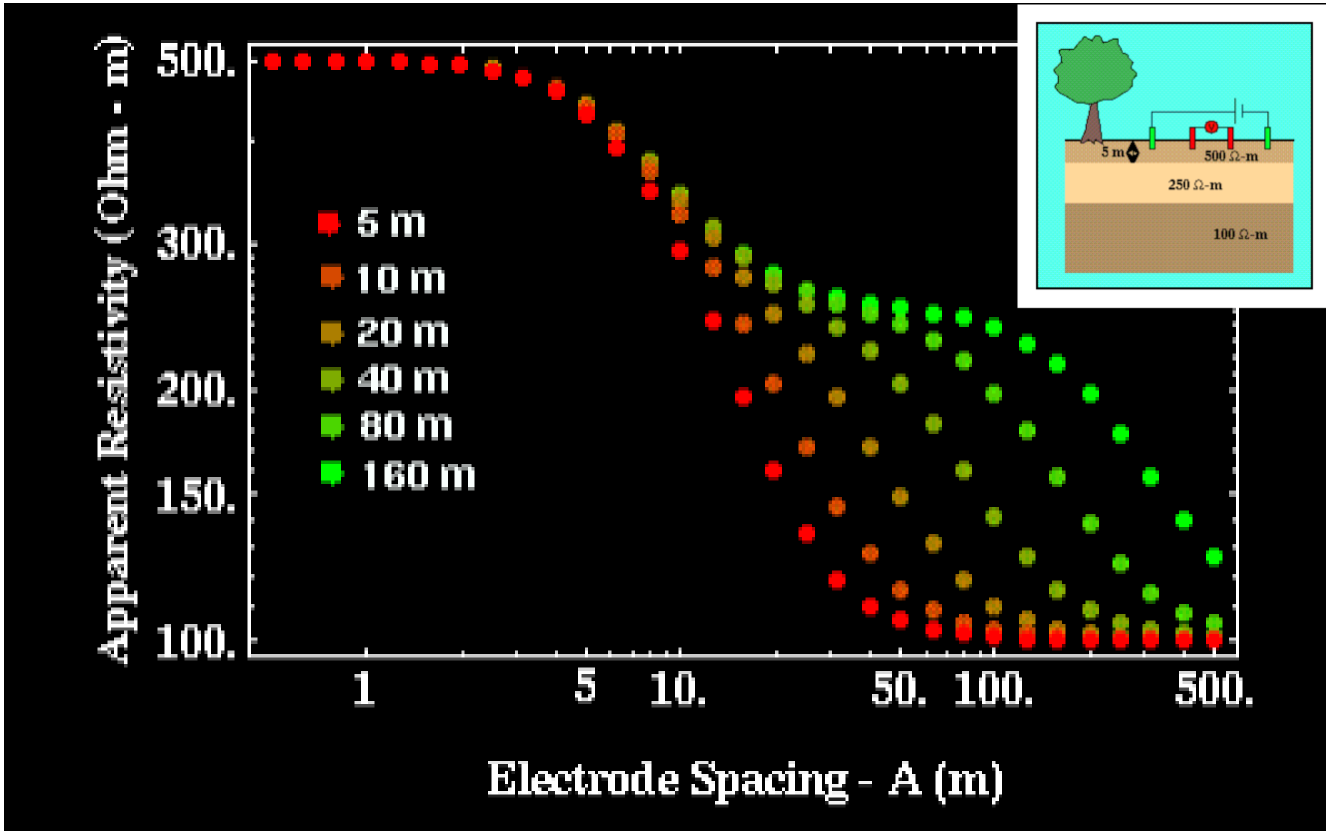


Layer over a half-space: ρ_{app} depends only on the ratios (ρ_2/ρ_1), and (a/d_1)

Two-layer over half-space: Apparent resistivity for the for various half-space resistivity values.



Two-layer over half-space: Dependence of apparent resistivity on the thickness of the middle layer.



May or may not pick up sandwiched layer depending on thickness, contrast

For 3+ layers:

Rule of thumb: If layer thickness < 0.1 the depth to top of layer, it cannot be resolved. Also, solution from sounding can be highly non-unique

But resolution also depends on resistivity contrast: thicker layers may not be resolved if contrast is too small; transition of apparent resistivity versus ***a-spacing*** is much sharper for a resistive layer over a conductive layer than for the opposite.